



Optimization of cutting conditions for single pass turning operations using a deterministic approach

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Abstract

An optimization analysis, strategy and CAM software for the selection of economic cutting conditions in single pass turning operations are presented using a deterministic approach. The optimization is based on criteria typified by the maximum production rate and includes a host of practical constraints. It is shown that the deterministic optimization approach involving mathematical analyses of constrained economic trends and graphical representation on the feed-speed domain provides a clearly defined strategy that not only provides a unique global optimum solution, but also the software that is suitable for on-line CAM applications. A numerical study has verified the developed optimization strategies and software and has shown the economic benefits of using optimization. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Cutting conditions; Machining; Turning; Optimization; Process planning

1. Nomenclature

A, A_1	constants	T	tool-life in time units
C_T	average production cost per component	T_{ac}	actual cutting time
D	workpiece diameter	T_c	feed engagement time
$f_F(V)$	f at the intersect ion of V and $\partial T_T / \partial f = 0$	T_L	workpiece loading and unloading time
F_P	power force	T_{max}, T_{min}	maximum and minimum tool-life limits
$F_{P_{max}}$	machine tool maximum power force limit	T_R	tool replacement time per tool failure
$f_{PT_{max}}$	f at the intersect of T_{max} and P_{max}	T_T	average production time per component
$f_{T_{max}}$	f at the intersection of V_{min} and T_{max}	V, f, d	cutting speed, feed per revolution, depth of cut
$f_{T_{min}}$	f at the intersection of V_{min} and T_{min}	V_a	V at which the P_{max} limit becomes effective
$K, n, n_1, n_2, \alpha, \beta, E, W$	constants	$V_{P(f)}$	V at the intersection of f and P_{max}
N	spindle rotational speed	$V_{PT_{max}}$	V at the intersect of T_{max} and P_{max}
P	machining power	$V_{T_{max}}$	V at the intersection of f_{up} and T_{max}
P_{max}	machine tool maximum power limit	$V_{T_{min}}$	V at the intersection of f_{up} and T_{min}
$R_{T_{max}}$	maximum surface roughness limit (peak-to-valley height)	$V_V(f)$	V at the intersection of f and $\partial T_T / \partial V = 0$
		x	labour and overhead cost rate
		y	tool cost per failure
		ℓ	workpiece length
		ψ_r, κ'_r	approach and minor cutting edge angles

2. Introduction

Machining is a major manufacturing process and plays a key role in the creation of wealth. It has been the driv-

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ing force for the generation and introduction of computer numerical control (CNC) and flexible automation in today's manufacturing. Due to the high level of automation and to offset the high capital and operating costs, it is estimated that modern manufacturing systems would use as high as 80% of the available production times on machining operations, as compared to about 5% in conventional manual machine tools [1]. This trend is encouraging and places a great demand to optimize the machining operations for further economic gains. As early as 1907, Taylor [2] recognized the existence of an optimum cutting speed in single pass turning. However, despite the considerable amount of research since Taylor's work, the progress in developing realistic optimization strategies for the various machining operations has been slow. This is partly due to the lack of machining performance information and equations, and partly due to the complex nature of the optimization problem.

Traditionally, the optimization of machining operations involves the selection of economic cutting conditions, such as the feed and cutting speed, according to a variety of economic criteria, such as the minimum production time and cost [3,4]. A realistic optimization study should also consider the many technological and practical constraints, which limit the feasible domain for the selection of optimum cutting conditions. This task has proven to be surprisingly difficult. It requires intricate mathematical analysis and computer assistance, and depends on quantitatively reliable mathematical functions for the machining performance measures (such as tool-life, power and surface finish) and detailed specifications of the machine tools, cutting tools and components, which act as constraints on the feasible cutting conditions [5–9]. This difficulty has resulted in some researchers using the available mathematical programming and numerical search techniques in attempts to provide the optimum feed and cutting speed in practical machining operations (e.g. Refs. [10,11]). These computer-packaged strategies neither ensure global optimum solutions nor provide clearly defined economic characteristics and solution strategies, which allow for the ready identification of trends in the way in which the optimum solution can change with alternative constraints. In addition, these purely numerical search approaches require excessive long computer processing time and the resulting strategies are not suitable for on-line application in CAM systems.

This study presents an on-going research aimed at developing realistic optimization strategies and CAM software for the various machining operations and for eventual integration into the Computer-Aided Process Planning (CAPP) system [12]. The optimization analysis, strategy and software module for the selection of cutting conditions in single pass turning on CNC machine tools are outlined and discussed. The analysis is based on the popular economic criterion of minimum

production time (or maximum production rate) while the resulting strategy applies for the minimum cost per components criterion due to the proven mathematical similarity of the two objective functions [5–9]. The constraints considered include the machine tool feed and speed limits, maximum power force, spindle torque and power constraints, the component surface roughness constraint, and the minimum and maximum tool-life limits that may be imposed by the production systems. A numerical study is presented to verify the developed optimization strategy and software and demonstrates the economic benefits of using optimization over handbook recommendations in turning operations.

3. Objective functions and constraints

Based on the maximum production rate (or the minimum average production time per component) criterion, the objective function for a single pass turning operation can be given by the equation [3]:

$$T_T = T_L + T_c + T_R \frac{T_{ac}}{T} \quad (1)$$

where the symbols are defined in the Nomenclature. The third term in the equation in fact represents the average tool replacement time per component. Similarly, the objective function for the average cost per component criterion is

$$C_T = x \left(T_L + T_c + T_R \frac{T_{ac}}{T} \right) + y \frac{T_{ac}}{T} \quad (2)$$

If a term T_R' is introduced, such that

$$T_R' = T_R + \frac{y}{x} \quad (3)$$

Eq. (2) becomes

$$C_T = x \left(T_L + T_c + T_R' \frac{T_{ac}}{T} \right) \quad (4)$$

It can be noted that if the labour and overhead cost rate, x , and the tool cost per failure, y , are minimized and constant through good management and purchasing policy, Eqs. (1) and (4) are mathematically similar. Hence, the characteristics and strategies for minimizing T_T and C_T are similar although the optimum feed and speed for the two criteria are not necessarily the same under the same constraint conditions. Thus, only the analyses for the minimum time per component T_T equation will be outlined in this article.

The tool-life is given in the typical extended Taylor-type equation

$$T = \frac{K}{V^{1/n} f^{1/n_1} d^{1/n_2}} \quad (5)$$

The cutting time T_c for turning of a length ℓ can be approximately expressed as:

$$T_c \approx T_{ac} = \frac{\pi D \ell}{V f} \quad (6)$$

Substituting Eqs. (5) and (6) into Eq. (1) gives

$$T_T = T_L + \frac{\pi D \ell}{V f} + \frac{\pi D \ell T_R}{K} V^{1/n-1} f^{1/n_1-1} d^{1/n_2} \quad (7)$$

This is the fundamental form of the objective function to be optimized. As is usual in single pass machining optimization studies, only the cutting speed V and feed f need to be optimized, since it is expected that the loading/unloading time T_L and tool replacement time T_R have been minimized using work study techniques and well-designed handling devices.

In practice, the cutting speed V and feed per revolution f must be selected to minimize T_T in Eq. (7) without violating any of the constraints. These constraints in fact limit the feasible domain of speed V and feed f and result in a constrained optimum T_T . For a single pass turning operation on a CNC machine tool, the machine tool limiting force, $F_{P_{max}}$, spindle torque, $T_{q_{max}}$, maximum power, P_{max} , as well as the feed and spindle speed boundaries (f_{min} , f_{max} , N_{min} , N_{max}) are considered. The component surface roughness requirement will be included in finishing operations. In addition, the minimum and maximum tool-life limits that may be imposed by the production systems are considered. These constraints can be expressed mathematically as follows.

3.1. Machine tool speed and feed boundary constraints

For CNC machine tools, any feed and spindle speed within the specified minimum and maximum limits may be considered to be available for the selection of optimum cutting conditions. Mathematically, these constraints are given by

$$\pi D N_{min} = V_{min} \leq V \leq V_{max} = \pi D N_{max} \quad (8)$$

$$f_{min} \leq f \leq f_{max} \quad (9)$$

3.2. Machine tool power force constraint

The power force limit is imposed by the machine tool mechanism, such as the spindle and tool post, and has to be constrained to within the machine tool maximum permissible loading. In addition, excessive machining forces will cause the machining system deformation affecting the component quality. Using the empirical power force equation in the Chinese handbooks [13,14], this condition can be expressed as:

$$F_p = E f^\alpha d^\beta \leq F_{P_{max}} \quad (10)$$

Thus, the maximum power force constraint will result in a feed limit, i.e.

$$f \leq f_F = \left(\frac{F_{P_{max}}}{E d^\beta} \right)^{1/\alpha} \quad (11)$$

3.3. Machine tool maximum power and torque constraints

The cutting conditions selected must satisfy the condition that the machining power is within the machine tool maximum power limit P_{max} . In the low-speed region of a machine tool operating range, the machine tool maximum power may not be permitted since this would involve an excessive spindle torque. In this region, the ‘low’ speed power constraint P_a due to the limiting spindle torque must be considered. This lower spindle power usually increases linearly with the speed until a critical speed V_a where the machine tool maximum power constraint P_{max} becomes relevant. Hence, the combined torque (or low-speed power P_a) and the maximum power constraints can be expressed as:

$$P = W V f^\alpha d^\beta \leq P_a = A_1 N = A V \quad (\text{for } V \leq V_a) \quad (12)$$

$$P = W V f^\alpha d^\beta \leq P_{max} \quad (\text{for } V > V_a) \quad (13)$$

It is common that V_a has a constant value (dependent on the workpiece diameter) between the minimum and maximum machine tool speed limits and can be found from the machine tool specification. At $V = V_a$, $P_a = P_{max}$ so that V_a can be found to be:

$$V_a = \frac{P_{max}}{A} \quad (14)$$

In addition, the low-speed power constraint can be represented by a feed limit f_a that can be found by rearranging Eq. (12) with $V = V_a$, $P_a = P_{max}$, i.e.

$$f \leq f_a = \left(\frac{A}{W d^\beta} \right)^{1/\alpha} \quad (15)$$

In contrast, the maximum power constraint P_{max} will limit both the feed and speed when Eq. (13) is satisfied.

3.4. Component surface roughness constraint

Although some theoretical surface roughness equations have been reported, there is a general lack of published data to support these equations so that this constraint cannot be accurately allowed for in the machining optimization until reliable surface roughness equations and associated data become available. For the purpose of the present work, the theoretical or ideal peak-to-valley height equation given in Ref. [3] will be employed. The resulting expression for surface roughness constraint is given by

$$f \leq f_{RT} = R_{T_{\max}}(\tan \psi_r + \cot \kappa'_r) \tag{16}$$

3.5. Minimum and maximum tool-life limits

It has become evident that under certain conditions, the optimum tool-life determined may be very small requiring a large number of cutting tools for replacement [5–8]. This is particularly so when using minimum production time criterion on CNC machine tools where the tool replacement time is very small. Although the optimum tool-life satisfies the selected economic criterion, it may be considered as impractical. Thus, a lower tool-life limit may be imposed by the production system either because of the tool supply or the limit of number of tools in the tool magazine. Similarly, it is not unreasonable that under some conditions, the production system may impose a tool-life range so that the cutting conditions selected should satisfy the tool-life limits. Consequently, the minimum and maximum tool-life limits are introduced in this study. These limits in fact specify a feasible feed and speed domain for the selection of optimum cutting conditions according to the tool-life equation (5).

In the above constraint equations, E , W , α and β are empirical constants dependent on the tool-work material combination; $F_{P_{\max}}$, P_{\max} , N_{\min} , N_{\max} , f_{\min} and f_{\max} are constraints given in the machine tool specifications; $R_{T_{\max}}$ is the maximum surface roughness (peak-to-valley) limit, and ψ_r and κ'_r are, respectively, the approach angle and minor cutting edge angle of the cutting tool.

It is evident that the magnitudes of the above constraints limit the permissible domain for the optimization of the cutting speed V and feed f in Eq. (7). Furthermore, the machine tool low-speed power (or torque) and power force constraints as well as the component surface roughness constraint, which only limit the permissible feed and are mutually exclusive, can be generalized by a feed limit f_x , i.e.

$$f \leq f_x = \min\{f_F, f_a, f_{RT}\} \tag{17}$$

For rough turning, Eq. (17) can be simplified as

$$f \leq f_x = \min\{f_F, f_a\} \tag{18}$$

The upper feed limits, f_x and f_{\max} , can be further generalized by:

$$f_{\text{up}} = \min\{f_x, f_{\max}\} \tag{19}$$

A detailed study of the machining performance data in the Chinese handbooks [13,14] has found that the exponents in the tool-life and the constraint equations have the following relationships: $1/n > 1/n_1 > 0$, $1/n > 1$ and $1 > \alpha > n/n_1$ while $1/n_1$ can be greater than, equal to or less than 1. The optimization analysis in this work will be developed based on these common relationships of the exponents.

4. Optimization analysis and economic characteristics

Mathematically, a global minimum time per component T_T requires that the partial derivatives of the objective function in Eq. (7) with respect to the cutting speed and feed are zero, i.e.

$$\frac{\partial T_T}{\partial V} = \frac{\pi D \ell}{V^2 f} \left[\frac{T_R}{T} \left(\frac{1}{n} - 1 \right) - 1 \right] = 0 \tag{20}$$

$$\frac{\partial T_T}{\partial f} = \frac{\pi D \ell}{V f^2} \left[\frac{T_R}{T} \left(\frac{1}{n_1} - 1 \right) - 1 \right] = 0 \tag{21}$$

Re-arranging these two equations will give the economic tool-life equations with respect to the cutting speed and feed, respectively, namely

$$\frac{K}{V^{1/n} f^{1/n_1} d^{1/n_2}} = T_R \left(\frac{1}{n} - 1 \right) = T_V \tag{22}$$

$$\frac{K}{V^{1/n} f^{1/n_1} d^{1/n_2}} = T_R \left(\frac{1}{n_1} - 1 \right) = T_F \tag{23}$$

To simultaneously satisfy Eqs. (20) and (21) (or Eqs. (22) and (23)) requires $n = n_1$. For common tool-work material combinations, $n \leq n_1$ so that a unique pair of V and f for a global minimum time per component T_T does not exist. Therefore, it is necessary to study the T_T characteristics in order to establish a strategy for selecting the V and f such that the production time per component is minimized.

Fig. 1 illustrates the $\partial T_T / \partial V = 0$ and $\partial T_T / \partial f = 0$ loci on an f - V graph. It has been proved [7–9] that for the usual values of the empirical tool-life equation exponents $1/n > 1/n_1 > 1$, the $\partial T_T / \partial f = 0$ curve is above and to the right of the $\partial T_T / \partial V = 0$ curve on the f - V diagram. Further, a local optimum T_T with respect to V always exists for a given f , since $1/n > 1$, which can be found on the curve described by Eq. (22), i.e. the $\partial T_T / \partial V = 0$ curve. Similarly, the optimum feed for

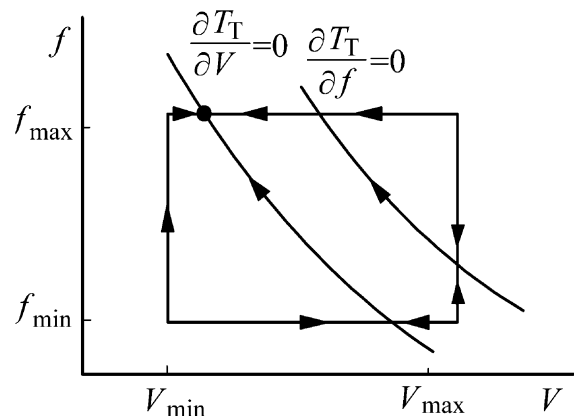


Fig. 1. T_T characteristics and feed and speed boundary constraints when $1/n > 1/n_1 > 1$.

a local minimum T_T can be obtained from Eq. (23) (on the $\partial T_T/\partial f = 0$ curve) for a given cutting speed V when $1/n_1 > 1$.

The time per component T_T characteristics along the $\partial T_T/\partial V = 0$ locus can be found by substituting the cutting speed V from Eq. (22) into Eq. (7), from which

$$T_T = T_L + \pi D \ell \left[\frac{T_R}{nK} \right]^n (1-n)^{n-1} f^{n/n_1-1} d^{n/n_2} \quad (24)$$

Thus, since $n/n_1 < 1$ or $(n/n_1-1) < 0$, T_T will decrease along the $\partial T_T/\partial V = 0$ curve as f increases (or V decreases), as indicated by the arrowheads in Fig. 1. It can also be proved that T_T along the $\partial T_T/\partial f = 0$ locus (when $1/n_1 > 1$) possesses similar characteristics to those of the $\partial T_T/\partial V = 0$ curve, as shown in Fig. 1.

However, when $1/n > 1$ but $1/n_1 \leq 1$, as is possible for some tool-work material combinations noted in the handbook [13,14], $\partial T_T/\partial f$ in Eq. (21) is negative and Eq. (23) does not apply. Thus, the necessary condition for a local minimum with respect to f (i.e. $\partial T_T/\partial f = 0$) can never be satisfied and the minimum T_T for a given V occurs when f is as high as possible. By contrast, Eqs. (20) and (22) still apply from which the optimum cutting speed can be found for a given f . It can be shown again that the T_T value decreases along the $\partial T_T/\partial V = 0$ locus as f increases (or V decreases).

The above T_T characteristics lead to the popular strategy of selecting V and f in the ‘high feed–low speed’ region in the vicinity of the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ (when $1/n_1 > 1$) loci. However, this strategy is not always valid in selecting the optimum cutting speed and feed, since in practice a number of technological and practical constraints, such as those noted above, have to be satisfied. The relevant groups of these constraints are considered subsequently before developing the optimization strategy allowing for all the constraints.

4.1. Effects of machine tool feed and speed boundary constraints

For CNC machine tools, the minimum and maximum feed limits define an available feed-speed domain with the upper and lower boundaries occurring at f_{\max} and f_{\min} , respectively. The minimum and maximum spindle speeds establish the cutting speed boundaries, for a given workpiece diameter, as vertical lines on the f – V diagram.

The T_T trends along the horizontal f_{\min} and f_{\max} boundaries as well as the vertical V_{\min} and V_{\max} boundaries can be established readily by superimposing the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ (when $1/n_1 > 1$) curves on an f – V diagram, as shown in Fig. 1. The T_T value reducing direction is again shown by the arrowheads on the feed and speed boundaries. Based on the T_T characteristics, the minimum T_T value along a feed boundary always occurs at its intersection with the $\partial T_T/\partial V = 0$ curve. Likewise, when $1/n > 1/n_1 > 1$ the minimum T_T on a

cutting speed boundary can be found at its intersection with the $\partial T_T/\partial f = 0$ locus. However, when $0 < 1/n_1 \leq 1$, the $\partial T_T/\partial f = 0$ locus does not exist and T_T will monotonically decrease along the constant V_{\min} and V_{\max} boundaries as f increases, according to the characteristics noted earlier. Combining these trends with the T_T characteristics along the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ loci will arrive at the required optimum solution.

It is apparent that the optimum solution depends on the relative positions of the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ (when $1/n_1 > 1$) loci with respect to the feed and cutting speed boundaries, which in turn depends on the input conditions, i.e. the magnitude and relationship of the machining performance data, the values of the limiting boundaries and the time factors in the T_T equation. From a detailed study of the T_T characteristics when only feed and cutting speed boundary constraints are considered, five possible optimum solutions have been identified and the corresponding limiting conditions established, Fig. 1 showing one of them, where the dot highlights the optimum solution.

4.2. Effects of machine tool force and power and component surface roughness constraints

The machine tool maximum power force, low-speed power (or spindle torque) and the component surface roughness constraints have been generalized and represented by an upper feed limit f_x . In the ‘high’ cutting speed region of a machine tool operating range, the machine tool maximum power constraint will come into play and limit both the feed and cutting speed from which a constrained optimum can be selected.

In order to establish an optimization strategy, it is again necessary to study the T_T trends on the f – V diagram while considering the effects of these constraints. As shown in Fig. 2, the maximum power P_{\max} and the $\partial T_T/\partial V = 0$ curves intersect such that for the usual set of exponent values, $1 > \alpha > n/n_1 > 0$, the slope of the P_{\max} curve is less than that of the $\partial T_T/\partial V = 0$ locus at the point of intersection and so the curves cross in the

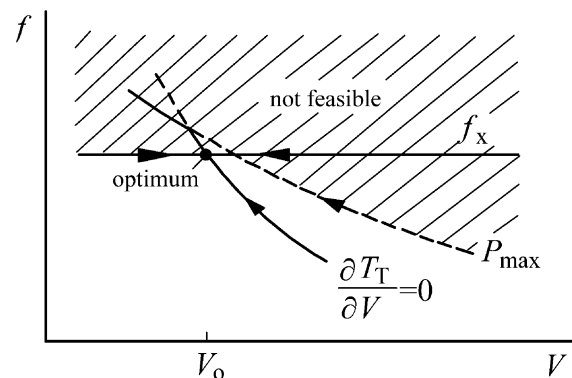


Fig. 2. T_T characteristics and force and power constraints.

way shown in the figure. It can be proven that the maximum power constraint curve intersects the $\partial T_T/\partial f = 0$ curve in the same manner as the $\partial T_T/\partial V = 0$ curve on the f - V graph.

The T_T trend along the P_{max} locus may be found by substituting V from Eq. (13) (with $P = P_{max}$) into Eq. (7). This shows that for the common conditions of $n/n_1 < \alpha < 1$ and $1/n > 1/n_1 > 0$, T_T decreases along the P_{max} locus as f increases. This is shown by the arrow-heads in Fig. 2. Thus, if a feed f is below the intersection of P_{max} and $\partial T_T/\partial V = 0$ curves, the optimum cutting conditions will be on the $\partial T_T/\partial V = 0$ locus; otherwise, the optimal solution will lie on the P_{max} curve as the cutting conditions on the $\partial T_T/\partial V = 0$ locus are not feasible. The portions of the P_{max} and $\partial T_T/\partial V = 0$ curves on which the optimum point is likely to lie are shown by solid lines in Fig. 2.

When both the machine tool power P_{max} and the generalized upper feed f_x constraints are considered jointly, the T_T characteristics can be found by superimposing the loci of these constraints on the f - V diagram as shown in Fig. 2. The resulting optimum solution will be on the generalized f_x constraint, at its intersection with either the P_{max} curve or the $\partial T_T/\partial V = 0$ curve, depending on which intersection is at a lower cutting speed.

As discussed in Eq. (19), the upper feed limit f_x resulting from the force, torque and surface finish constraints and the maximum feed constraint f_{max} are further generalised by an upper feed limit f_{up} . The T_T characteristics when jointly considering P_{max} and f_{up} are the same as those shown in Fig. 2, i.e. when f_{up} is lower than the feed in the intersection of P_{max} and $\partial T_T/\partial V = 0$ curves, the optimum is at the intersection of f_{up} and $\partial T_T/\partial V = 0$, $(V_V(f_{up}), f_{up})$; otherwise it is at the intersection of f_{up} and P_{max} , $(V_P(f_{up}), f_{up})$, where $V_V(f)$ and $V_P(f)$ are, respectively, the cutting speeds on the $\partial T_T/\partial V = 0$ and P_{max} curves when $f = f_{up}$, and are given by

$$V_V(f) = \left[\frac{K}{T_R(1/n-1)f^{1/n_1}d^{1/n_2}} \right]^n \quad (25)$$

$$V_P(f) = \frac{P_{max}}{Wf^\alpha d^\beta} \quad (26)$$

As discussed earlier, the cutting speed V_a at the intersection of P_{max} and torque (or lower speed power) constraint is between V_{min} and V_{max} , and the feed corresponding to V_a at this intersection is f_a . Since $f_{up} = \min\{f_F, f_a, f_{RT}, f_{max}\}$ and the P_{max} curve is decreasing as f decreases and V increases in the f - V domain, it can be proven from Eq. (26) that cutting speed $V_P(f_{up})$ at the intersection of f_{up} and P_{max} is always greater than the machine tool minimum cutting speed limit V_{min} .

4.3. Minimum and maximum tool-life constraints

The minimum and maximum tool-life limits, T_{min} and T_{max} , define a feasible feed-speed domain for the selection of cutting conditions, as shown in Fig. 3. For given values of T_{min} and T_{max} , these limits have the same mathematical form as Eqs. (22) and (23), and display in the same manner as the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ curves in the f - V diagram. Similar to Eq. (24) for the $\partial T_T/\partial V = 0$ curve, it can be proven that T_T decreases along the T_{min} and T_{max} curves as f increases or V decreases. Under different input conditions, the T_{min} and T_{max} curves shift in the f - V domain with respect to feed and cutting speed boundaries, the P_{max} and f_{up} limits and the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ curves, resulting in different possible optimum solutions, as will be identified subsequently.

5. Optimization strategy for CNC machine tools

The above study has separately considered small groups of most related constraints. In practical situations, the economic trends and combined effects of all the constraints have to be considered when machining on CNC machine tools. This results in a more complex strategy, which benefits greatly from computer assistance for its implementation after the various possible constrained optimum solutions and the corresponding limits for identifying these solutions are established.

By applying the above analyses, the economic trends for the combined effects of all the constraints can be found by superimposing the upper feed limit, f_{up} , and the maximum power P_{max} limit, the minimum and maximum cutting speed and f_{min} boundaries, the minimum and maximum tool-life limits as well as the $\partial T_T/\partial V = 0$ and $\partial T_T/\partial f = 0$ loci on the f - V diagram. Since the relative positions of these curves on the f - V diagram can vary depending on the magnitudes of the constraints, the machining performance data and the cut geometry (depth

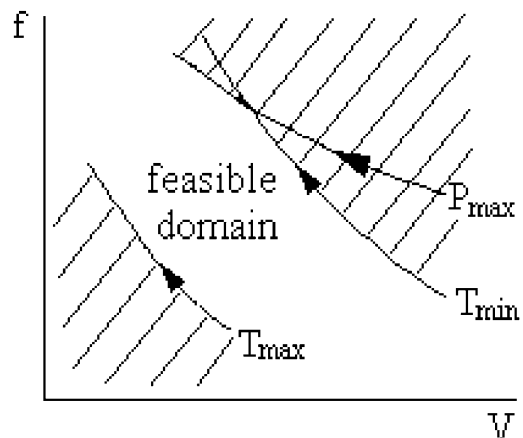


Fig. 3. T_T characteristics and tool-life constraints.

of cut), the active constraints on which the optimum conditions may lie, can also vary. To establish the optimization strategy, it is necessary to identify the various constrained optimum solutions on the ‘active’ constraints, and the associated limiting conditions for each solution.

A detailed mathematical study of the T_T trends on the f – V domain has resulted in 12 distinctly different solutions representing all possible relative positions of the loci of $\partial T_T/\partial V = 0$, $\partial T_T/\partial f = 0$ and the various constraints. It is noteworthy that among the 12 possible solutions, one that can occur in one of the five different cases represents the situation where a single pass operation is not feasible since at least one of the practical constraints will be violated for the ‘input’ conditions. For such a case, a multipass operation or an alternative machine tool must be considered.

The various possible optimum solutions are shown in Fig. 4 where the arrowheads indicate the T_T decreasing direction and the ‘dot’ highlights the optimum feed and cutting speed. The limiting conditions for identifying each constrained optimum f and V solution are given in the captions of each diagram, based on which a flowchart, as shown in Fig. 5, has been developed for computer programming to arrive at the required constrained global optimum f and V solution for a minimum T_T (or C_T). The inputs required are machine tool constraints (f_{\min} , f_{\max} , N_{\min} , N_{\max} , $F_{P_{\max}}$, P_{\max} , and N_a or V_a), component surface requirement ($R_{T_{\max}}$), tool-life limits (T_{\min} and T_{\max}), cut geometry (d , ℓ), time and cost (if C_T criterion is considered) factors (T_L , T_R , x , y), and machining performance information in the force, power and tool-life equations. The computer program has been tested and debugged for each possible branch or solution according to the flowchart. Despite the complexity of the constrained optimization strategy, the use of computers has assisted its implementation.

6. Numerical assessment of the optimization model

In order to validate the optimization strategy and computer program and to assess the benefit of using optimization in process planning, a numerical study has been carried out. It is believed that the cutting conditions selected in workshops are mostly based on the experience of the operators or process planners, and are not consistent. Therefore, the recommendations from machining data handbooks have been adopted for this purpose. It is noted that many machining data handbooks [15] only provide recommendations for the selection of some of the cutting conditions, such as the feed and cutting speed in turning, irrespective of the machine tool (constraints) used. However, it has been found that a few comprehensive cutting conditions handbooks [13,14] provide not only complete tool-life, force, torque and power equations for a range of tool-work material com-

binations, but also detailed information and specifications for a number of machine tools. The equations for these cutting performance measures apply to a variety of tool-work material combinations, and only the values of the constants in the equations need to be changed for different tool and work materials. Furthermore, these handbooks provide a methodology for selecting a standard cutting tool as well as the feed and speed for single pass turning such that the conventional machine tool constraints are not violated. A study of this methodology clearly indicated that the handbook solutions were feasible, although not necessarily optimal [5,7]. Nevertheless, these handbooks provide a unique opportunity to assess the developed optimization strategies and software as the optimized times and costs should always be equal or superior to those from the handbooks. Further, the benefits of using optimization strategies over handbook recommendations can be evaluated by a numerical study.

For this numerical study, rough turning operations were simulated on a conventional lathe found in the handbooks [13,14] where the discrete feed and speed steps were ignored to simulate a CNC machine tool. The relevant machine tool specifications or limitations are given in Table 1. A carbide cutting tool was used to cut a plain carbon steel ($\leq 0.6\%C$) of ultimate tensile strength $\sigma_b = 657\text{MPa}$. The cutting performance data for this tool-work combination are given in Table 1. This cutting tool had a major cutting edge angle of 45° while the other major tool angles (cutting edge inclination and normal rake angles) fall in the range of 5 and 10° and the cutting performance data in Table 1 are applicable to this range.

Three levels of depths of cut d spanning the handbook recommended ranges and three levels of workpiece diameters D were tested at three levels of component lengths ℓ , three levels of loading/unloading time T_L and three levels of tool replacement time T_R . When the minimum cost per component criterion was considered, two labour and overhead cost rates x were selected, i.e. at \$1.00 and \$1.50/min, with the tool cost per failure (per cutting edge) y at \$4.00. As tool-life limits are not considered in the handbooks, in order to facilitate the comparisons between the handbook and optimized solutions, tool-life limits were given two extreme values so that these constraints are not effective. Detailed values for these parameters are given in Table 2. Thus 243 combinations were considered for the minimum time per component criterion and 486 for the minimum cost criterion.

Examining the optimum solutions for both minimum T_T and C_T criteria has revealed that for all cases studied, the optimum feeds and cutting speeds are considered as feasible for rough turning operations. It is noted that for all the combinations or cases, the optimum feeds range from 0.2 to 0.35 mm while the optimum cutting speeds

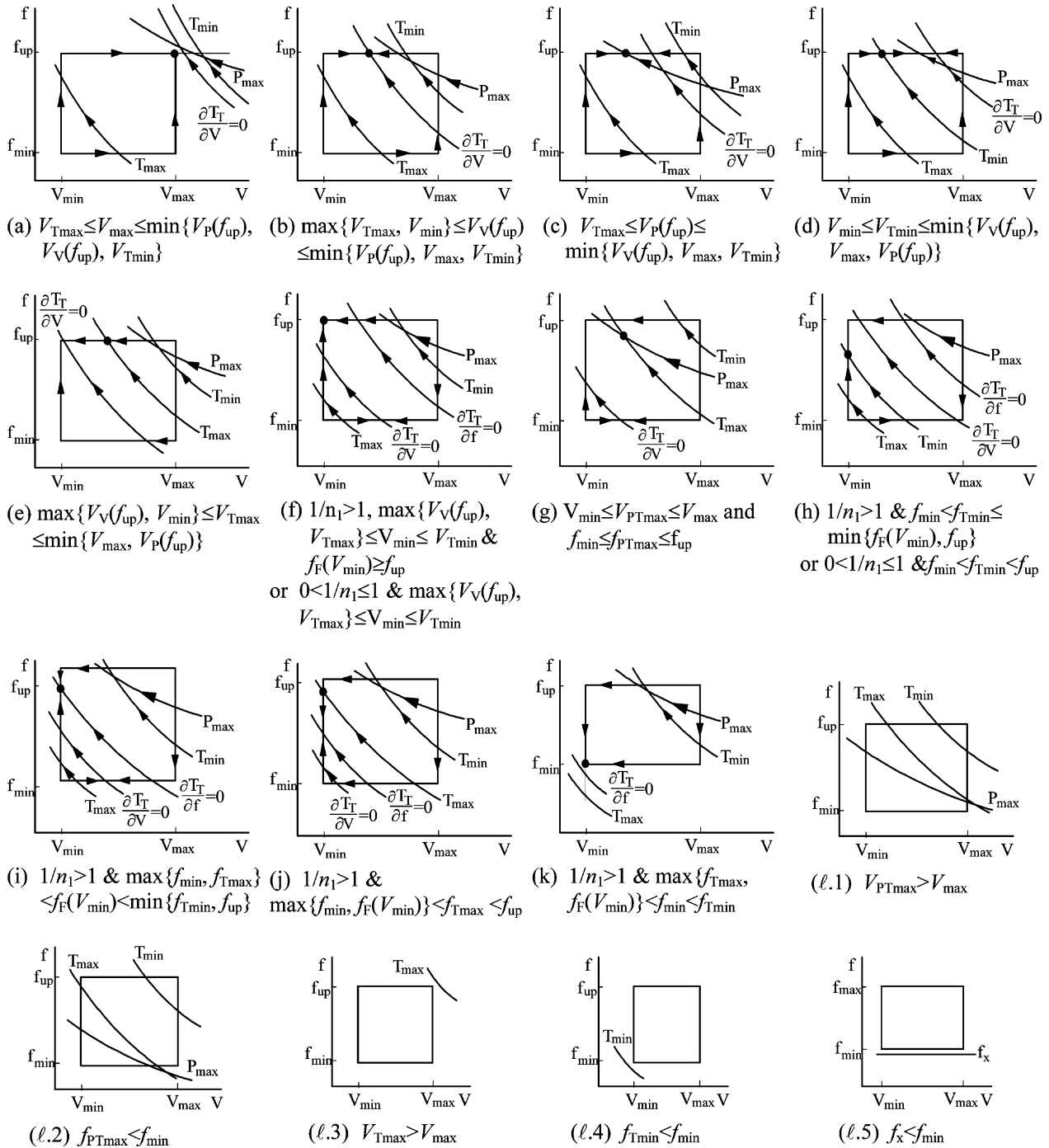


Fig. 4. Various possible constrained optimum solutions for single pass turning on CNC machine tools.

are from 99 m/min or 1.65 m/s to 137 m/min or 2.28 m/s (or the spindle speeds range from 158 to 440 rpm). The small variation of the optimum conditions are attributed to the small range (or increments) of the depth of cut and the time parameters used. Furthermore, it has been found that the optimum T_T and C_T for all the cases are superior or equal to those from the handbook recommendations [13,14], as shown in Fig. 6, where the

time and cost for each case were evaluated by using the handbook recommended feed and cutting speed based on Eq. (7).

In order to assess the benefit of using optimization over handbook solution in process planning, quantitative comparisons have been carried out between the handbooks [13,14] recommended and optimized solutions based on the equations below:

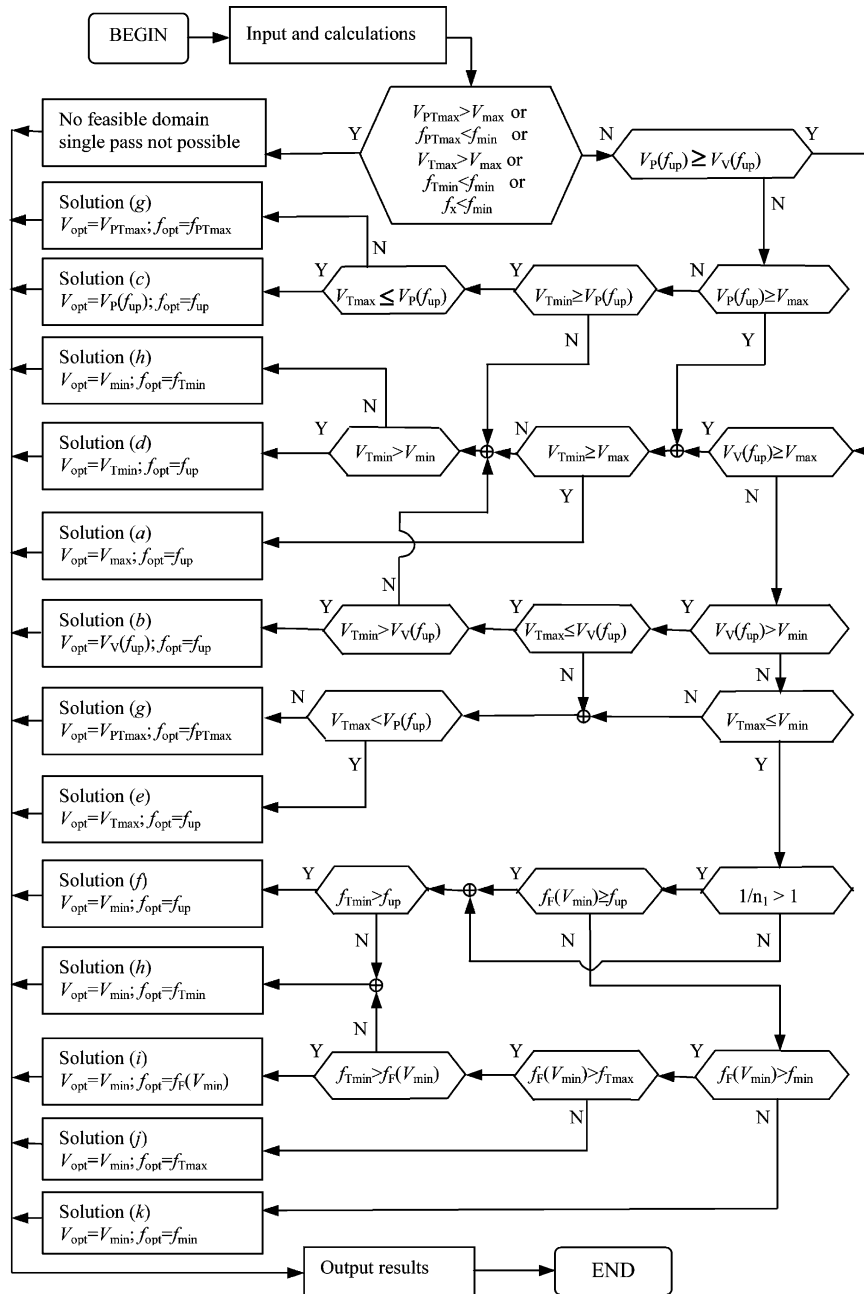


Fig. 5. Flowchart solution of the optimization strategy for single pass turning operations.

Table 1

Machine tool specifications and machining performance data for turning carbon steel with a carbide cutting tool

Machine tool specifications	$N_{\min} = 11.5\text{rpm}$ $N_{\max} = 1200\text{rpm}$ $N_a = 46\text{rpm}$	$f_{\min} = 0.082\text{mm/rev.}$ $f_{\max} = 1.590\text{mm/rev.}$	$P_{\max} = 7800\text{W}$ $\eta = 0.75$ $F_p = 3530\text{N}$
Machining performance data	Tool-life equation $n = 0.2, n_1 = 4.0, n_2 = 1.0,$ $K = 2.086\text{E}12$	Force and power equations $\alpha = 0.75, \beta = 1.0, E = 2795, W = 46.583$	

Table 2
Workpiece, time and cost parameters used in the numerical study

D (mm)	ℓ (mm)	T_L (min)	T_R (min)	d (mm)	x (\$/min)	y (\$)
100	200	1.0	0.3	2.0	1.00	4.00
150	400	3.0	0.6	3.0	1.50	
200	600	5.0	0.9	4.0		

$$\frac{\Delta T_T}{T_{To}} 100 = \left[\frac{T_{Tr} - T_{To}}{T_{To}} \right] 100 = \left[\frac{T'_{Tr} - T'_{To}}{T'_{To}} \right] \left[1 - \frac{T_L}{T_{To}} \right] 100 \quad (27)$$

$$\frac{\Delta C_T}{C_{To}} 100 = \left[\frac{C_{Tr} - C_{To}}{C_{To}} \right] 100 = \left[\frac{C'_{Tr} - C'_{To}}{C'_{To}} \right] \left[1 - \frac{xT_L}{C_{To}} \right] 100 \quad (28)$$

where the T'_T and C'_T are the time and cost per component when T_L is zero.

The histograms in Fig. 6 show the overall economic benefits of using optimization over handbook recommendation found in this simulation study. It is noted that the average percentage time increase or penalty of using handbook recommendation is about 15.7% with a range of about 1–55% while the corresponding cost penalties for $x = \$1.00/\text{min}$ are about 14.5% on average, ranging from about 1 to 48%. When the overhead and labour cost rate $x = \$1.50/\text{min}$ is used, similar scales of cost penalties for using handbook recommendations were noted. It is apparent that the penalties of using handbook cutting conditions are considerable, so that in general substantial benefits would be gained if these cutting conditions were optimized using the above strategies.

From the linearity of Eqs. (27) and (28), the maximum penalties of using handbook recommendations (or maximum benefits of using the optimum conditions) will occur when the loading/unloading and idle time T_L are

as small as possible (ideally zero). Also these penalties reduce linearly to zero as T_L/T_{To} and xT_L/C_{To} increase to 1. Thus, the economic benefits of using optimization in this numerical study can accordingly be increased when the loading/unloading time is reduced in modern flexible manufacturing systems, where the non-productive times and costs are minimized and are small proportions of the total production times and costs. Consequently, the use of optimization strategy in process planning is more important than in the past.

In order to assess the feasibility of using the developed optimization model and program for on-line application in computer-aided manufacturing systems, such as for adaptive control where the machining conditions are continually updated and the corresponding optimum cutting conditions are to be determined, the computer processing times have been recorded during the course of implementing the optimization strategies. When the program was run on a personal computer (Pentium III 850MHz CPU), the processing (excluding input and output) times for all the combinations of the selected conditions and for both the time and cost criteria were less than 0.01 s. Therefore, the developed deterministic, rather than numerical search, optimization strategies and software module are suitable for on-line applications in computer-aided manufacturing systems.

7. Conclusions

Using a deterministic optimization approach, a realistic optimization strategy for single pass turning on CNC machine tools has been presented. This optimization study is based on the criteria typified by the minimum production time per component while allowing for the many practical constraints. The detailed optimization analysis assisted by the feed-speed diagrams has provided an in-depth understanding of the economic characteristics and the influence of the constraints and machining performance data, which has resulted in a clearly defined optimization strategy that ensures the global

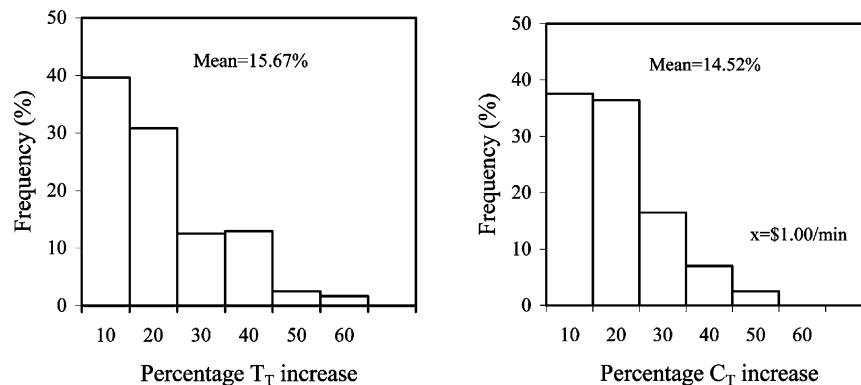


Fig. 6. Percentage production time and cost increase of handbook recommendations over optimized solutions.

optimum solution. The numerical study has validated the developed optimization strategy and computer program. It has also shown the substantial benefits in production time and cost per component that can be achieved when using the optimized cutting conditions rather than handbook recommendations. In addition, the study has demonstrated the suitability of the developed computer program for on-line applications in computer-aided manufacturing systems.

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