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PREDICTION OF CUTTING FORCES AND CHIP GEOMETRY IN OBLIQUE MACHINING FROM FLOW STRESS PROPERTIES AND CUTTING CONDITIONS

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Abstract—An approximate theory of oblique machining is given in which the mean friction angle previously used to describe the frictional condition at the tool–chip interface is replaced by the shear strength of the chip material at the tool–chip interface. The shear strength is expressed as a function of strain-rate and temperature using a velocity-modified temperature. Good agreement is shown between theory and experiment.

NOMENCLATURE

AB	shear plane
C	constant in strain-rate equation
F	frictional force at tool–chip interface
F'	component of F in the normal plane
F_C	force component in the direction of cutting
F'_C	component of F_c in the normal plane
F_s	force along AB
F'_S	component of F_S in the normal plane
$F_T = F'_T$	force component normal to the direction of cutting and machined surface.
F_R	force component normal to F_C and F_T
h	tool–chip contact length
i	inclination angle
J	mechanical equivalent of heat
K	thermal conductivity
k	shear flow stress
ℓ	length of AB
N	normal force at tool–chip interface
n	strain-hardening index
P_f	component of F normal to the normal plane
P_s	component of F_S normal to the normal plane
R	resultant cutting force
R'	component of R in the normal plane
R_T	thermal number
S	specific heat
T	temperature
T_{AB}	temperature along plane AB
T_{int}	temperature at tool–chip interface
T_M	maximum temperature rise at tool–chip interface
T'_M	maximum temperature rise at tool–chip interface when only F' is considered.
k_{chip}	shear flow stress in chip
T_{MOD}	velocity modified temperature
T_{SZ}	mean temperature rise in shear zone
T_W	initial work temperature
t_1	undeformed chip thickness
t_2	chip thickness
U	cutting velocity
U'	component of U in the normal plane
U''	component of U normal to the normal plane
V	chip velocity
V_S	shear velocity
V'_S	component of V_S in the normal plane
w	width of cut
w'	equivalent width of cut
α	tool rake angle

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α_n	normal rake angle
β	proportion of generated heat conducted into work
γ	shear strain
γ_{AB}	shear strain along AB
$\dot{\gamma}_{AB}$	shear strain-rate along AB
$\dot{\gamma}_{int}$	shear strain-rate along tool–chip interface
δ	ratio of thickness of plastic zone at tool–chip interface to chip thickness
ε	uniaxial strain
$\dot{\varepsilon}$	uniaxial strain-rate
$\dot{\varepsilon}_{AB}$	strain-rate along AB
η_c	chip flow angle
θ	angle between R and shear plane
θ_n	angle between R' and shear plane measured in the normal plane
λ	mean friction angle
λ_n	mean normal friction angle
ν	constant in velocity-modified temperature
ρ	density
σ	uniaxial stress
σ_1	flow stress at strain $\varepsilon = 1$
σ_{int}	flow stress along interface
ϕ	shear angle
ϕ_n	normal shear angle
τ_{int}	shear stress at tool–chip interface
τ'_{int}	interface shear stress when only considered in the normal plane
δ_c	direction of resultant frictional force (Fig. 5)
η_c	chip flow angle
δ_s	direction of resultant shear force
η_s	shear flow angle

INTRODUCTION

IN A PREVIOUS paper [1] an approximate theory of oblique machining (Fig. 1) has been developed by assuming that the plastic deformation in the plane normal to the cutting edge referred to as the normal plane is equivalent to the flow in orthogonal machining. It also shows how the chip geometry, the direction of chip flow, and the three components of cutting forces can be predicted from work material properties and cutting conditions. Good agreement has been shown with experimental results. A weakness of this approach is that the friction angle λ_n (subscript n refers to normal plane values) must be known before predictions of cutting forces, etc. can be made. In practice, λ_n cannot be taken as given information as it cannot be measured independently of machining tests and varies markedly with changes in cutting conditions.

In an analysis of orthogonal machining, Oxley *et al.* [2–5] have shown how λ_n can be replaced by the shear strength of the chip material at the tool–chip interface as a friction parameter. In this way the predictive value of their theory has been greatly enhanced. This approach has been adopted in the present work for analysing the mechanics in the normal plane and extended to predict the shear angle, the direction of chip flow and the three components of cutting forces in oblique machining.

ORTHOGONAL MACHINING THEORY

Investigations of the mechanics of machining are usually limited to the relatively simple case of orthogonal machining where a single straight cutting edge is set parallel to the surface being machined and normal to the cutting velocity. This is a special case of oblique machining (Fig. 1) with the inclination angle i equal to zero. (Inclination angle i (Fig. 1) is the angle between the cutting velocity and the normal to the cutting edge measured in the plane of the machined surface.)

In the analysis of orthogonal machining (Fig. 2) the process is usually assumed to be steady state with the chip formed by plastic deformation with no cracks occurring in the deforming material and with no build up of material on the cutting tool edge. It is also assumed that the deformation is plane strain with planes normal to the cutting edge being typical planes of flow. With the orthogonal model shown in Fig. 2, Stevenson and Oxley [4] have shown that the strain rate distribution is very nearly symmetrical about AB with the maximum value

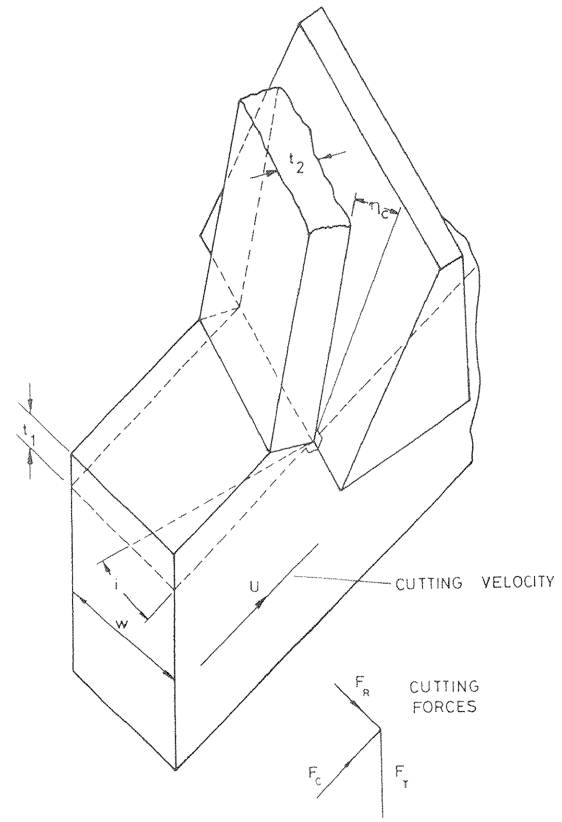


FIG. 1. Model of oblique machining process.

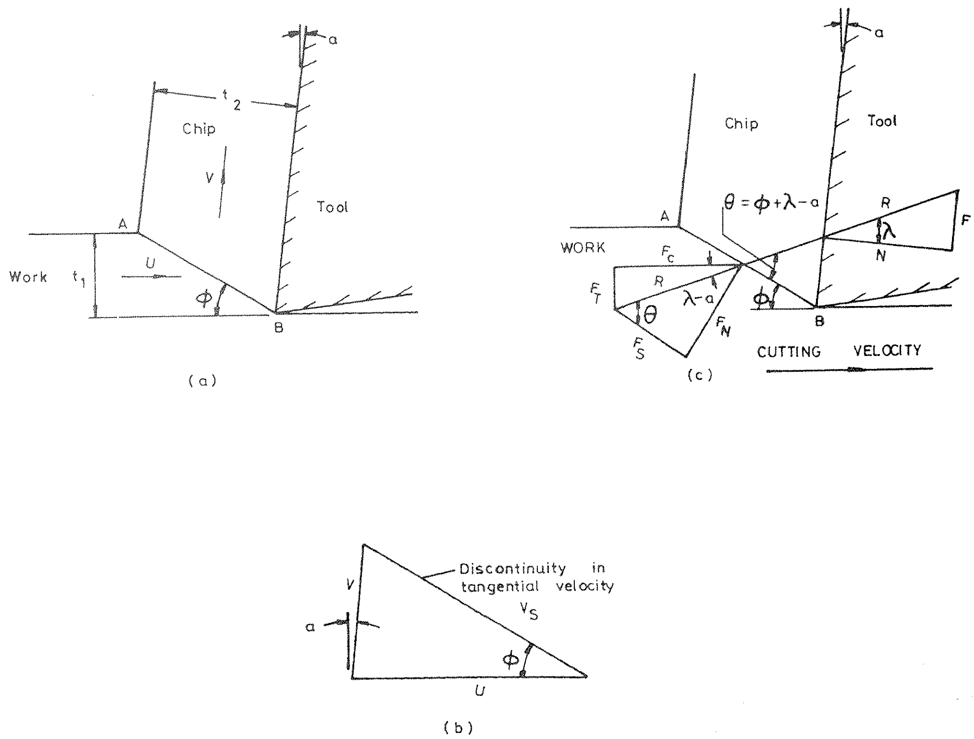


FIG. 2. Model of orthogonal machining process.

occurring at AB. This means approximately half the total strain has occurred at AB and that the shear strain along AB is given by

$$\gamma_{AB} = \frac{1}{2}\frac{\cos\alpha}{\sin\phi\,\cos(\phi-\alpha)} \tag{1}$$

where α (Fig. 2) is the tool rake angle and ϕ is the so called shear angle.

The mean value of the shear strain rate along AB can be estimated [4] from the empirical equation

$$\dot{\gamma}_{AB} = \frac{CV_s}{\ell} \tag{2}$$

where C is a material constant, V_s (Fig. 2) is the shear velocity and $\ell = t_1/\sin\phi$ is the length of AB with t_1 the undeformed chip thickness.

The shear stress on plane AB is the maximum shear flow stress and the normal stress is the hydrostatic stress. By analysing these stresses along AB it can be shown that

$$\tan\theta = 1 + 2\left(\frac{\pi}{4} - \phi\right) - Cn \tag{3}$$

where θ (Fig. 2) is the angle between the resultant cutting force R and AB, C is the constant in equation (2) and n is the strain-hardening index in the empirical stress/strain relation

$$\sigma = \sigma_1\varepsilon^n \tag{4}$$

where σ and ε are the uniaxial flow stress and strain and σ_1 and n are material constants which will in general be functions of strain-rate and temperature. By considering the force diagram in Fig. 2 it can also be shown that

$$\theta = \phi + \lambda - \alpha \tag{5}$$

where λ is the mean friction angle along the tool–chip interface defined by

$$\tan\lambda = \frac{F}{N} \tag{6}$$

where F and N are the friction and normal forces at the tool–chip interface.

In relating uniaxial flow stress σ, strain ε and strain-rate $\dot{\varepsilon}$ to the plane strain shear flow stress k, strain γ and strain-rate $\dot{\gamma}$, the following relations based on the shear strain energy yield criterion are used:

$$\sigma = \sqrt{3}\,k$$
$$\varepsilon = \frac{\gamma}{\sqrt{3}} \tag{7}$$
$$\dot{\varepsilon} = \frac{\dot{\gamma}}{\sqrt{3}}\ .$$

The work material used in the present study is S1214 steel (Australian specification) which is a free machining steel with the chemical composition of 0.084 % C, 0.28 % S, < 0.01 % Si, 0.064 % P, 0.94 Mn, and < 0.1 % Cu. The flow stress parameters σ_1 and n for the work material can be found from machining test results [1] as follows. Experimental values of ϕ and λ are used with equations (3) and (5) to calculate n with the corresponding strain rates ($\dot{\varepsilon} = \dot{\gamma}_{AB}/\sqrt{3}$) found from equation (2), C being taken as 5.8 which is the value found in a previous einvestigation [4] for a similar steel. σ_1 is then found from the equation

$$\sigma_1 = \frac{\sigma_{AB}}{\varepsilon_{AB}^n} = \frac{\sqrt{3}\,k_{AB}}{\left(\dfrac{\gamma_{AB}}{\sqrt{3}}\right)^n} \tag{8}$$

where γ_{AB} is given by equation (1) and k_{AB} is calculated from the experimental shear force F_s (Fig. 2), that is

$$k_{AB} = \frac{F_s \sin\phi}{t_1 w}$$

with F_s given by

$$F_s = (F_C^2 + F_T 2)^{1/2} \cos\theta \tag{9}$$

where F_C is the cutting force component in the cutting velocity direction and F_T the force normal to this direction. The calculated values of σ_1 and n are plotted against strain-rate in Fig. 3. These curves can be represented by the equations

$$\sigma_1 = 73.3 + 10.1 \log \dot{\varepsilon} \tag{10}$$

and

$$n = 0.39 - 0.04 (\log \dot{\varepsilon})^2 + 0.006 (\log \dot{\varepsilon})^3. \tag{11}$$

In orthogonal machining the temperature along AB can be found from

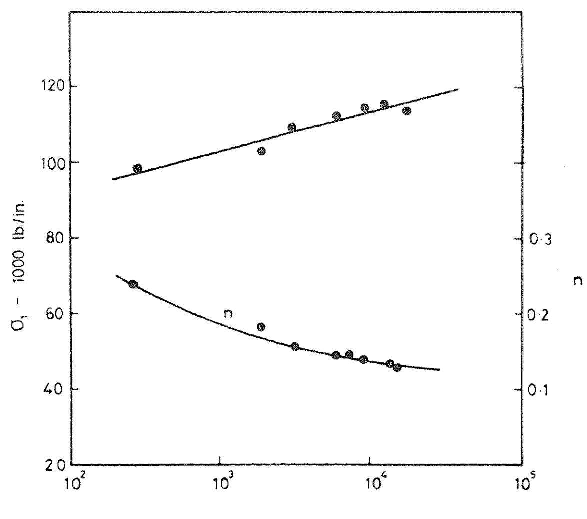
$$T_{AB} = T_{SZ} + T_w \tag{12}$$

where T_w is the initial work temperature and T_{SZ} can be estimated by considering the plastic work done in the shear zone, which gives

$$T_{SZ} = \frac{1-\beta}{12 J \rho S t_1 w}\left(F_C - \frac{F \sin\phi}{\cos(\phi - \alpha)}\right) \tag{13}$$

where ρ is the density of the work material (0.28 lb/in^3), S its specific heat (0.12 c.h.u./ib °C) and β is the proportion of heat conducted into the work, which can be found from the empirical equations based on experimental data given by Boothroyd [6]:

$$\begin{aligned} &\beta = 0.6 \text{ for } R_T \tan\phi < 0.5 \\ &\beta = 0.5 - \frac{0.5}{\log 40} \log(R_T \tan\phi) \\ &\qquad \text{for } 0.5 \leqq R_T \tan\phi \leqq 40 \\ &\beta = 0 \text{ for } R_T \tan\phi > 40. \end{aligned} \tag{14}$$



Where R_T is a thermal number given by

$$R_T = \frac{720\rho S U t_1}{K} \tag{15}$$

with K the thermal conductivity of the work material (2.1 c.h.u./in. hr °C).

The mean temperature along the tool–chip interface, T_{int} can be calculated from

$$T_{int} = T_{AB} + \frac{T_M}{2} \tag{16}$$

where T_M is the maximum temperature rise along the interface and is given by

$$T_M = \frac{1.13\,\tau_{int}}{J}\left(\frac{5Uh\sin\phi}{\rho SK\cos(\phi - \alpha)}\right)^{1/2} \tag{17}$$

where τ_{int} is the mean maximum shear stress along the interface and can be calculated from

$$\tau_{int} = \frac{F}{hw} \tag{18}$$

where F is the frictional force, h is the tool–chip contact length and w is the width of cut. Equation (15) was derived by Rapier [7] by considering the frictional work done along the tool–chip interface.

When predicting shear angles and cutting forces in orthogonal machining, Fenton and Oxley [2, 3] showed that shear flow stress in the chip, k_{chip}, varies with both temperature and strain-rate. They adopted the concept of a velocity-modified temperature and combined these parameters in an equation of the form

$$T_{mod} = T_{int}\left(1 - v\log\frac{\dot{\gamma}_{int}}{\dot{\gamma}_0}\right) \tag{19}$$

where T_{int} is the chip interface temperature, $\dot{\gamma}_0$ and v are constants. $\dot{\gamma}_{int}$ is the shear strain-rate at the interface and can be found from an empirical equation used by Fenton and Oxley [2, 3], that is

$$\dot{\gamma}_{int.} = \frac{V}{\delta t_2} \tag{20}$$

where V is the chip velocity and δt_2 is the width of the secondary shear zone with the value of δ taken as 1/10.

Values of k_{chip} can be calculated from equation (18) using experimental values of F and h. Corresponding values of T_{mod} can be calculated from equation (19) with v and $\dot{\gamma}_0$ taken as 0.018 and 0.001, respectively. The results of k_{chip} versus T_{mod} are plotted in Fig. 4.

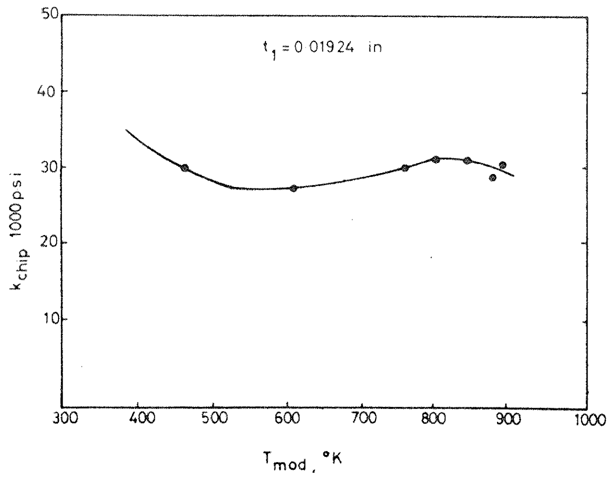


FIG. 4. Relation of k_{chip} and velocity-modified temperature.

NEW OBLIQUE MACHINING THEORY

To obtain a theory of oblique machining, one possible approach is to assume that the flow in the plane normal to the cutting edge can be analysed as if it is orthogonal (plane strain) flow. This is clearly an approximation as there will be flow normal to this plane, that is parallel to the cutting edge. The smaller the inclination angle i the better this approximation should be. An alternative approach would be to consider the flow in the plane containing the cutting velocity and chip velocity as equivalent to orthogonal flow. In this case, there is no flow normal to the plane but the normal stress on the shear plane does not act in a direction parallel to the plane. Preliminary trials using both approaches showed that the analysis based on the normal plane was the more promising.

Based on the above assumption, the oblique machining processes can be divided into two simultaneous steps: (i) orthogonal cutting in the normal plane, and (ii) a sliding process between the tool and the chip and a shearing process in the shear plane, both in a direction parallel to the cutting edge. As shown in Fig. 5a, the cutting velocity U can be resolved into two components, $U'(= U\cos i)$ and $U''(= U\sin i)$ normal and parallel, respectively, to the cutting edge. Thus the cutting process can be considered in such a way that the workpiece approaches the cutting tool with a cutting velocity U' and at the same time slides with a velocity U''. The three mutually perpendicular force components F_C, F_T, and F_R (Fig. 1) can be resolved into a component R' in the normal plane and a component P_f (or P_s) normal to this plane as shown in Fig. 5. R' will be the resultant cutting force required for step (i) and P_f (or P_s) the forces for step (ii). When considering the shear plane, the shear force required for step (i) will be Fs' while step (ii) requires the force P_s. It is now assumed that the orthogonal analysis given in the previous section applies to the normal plane with the resultant force R' replacing R. If w is the width of cut, then $w' = w/\cos i$ for step (i). The undeformed chip thickness ($t'_1 = t_1$) and the chip thickness ($t'_2 = t_2$) are unchanged.

Considering the force diagram in the normal plane (Fig. 5b), the frictional force component F' is given by

$$F' = F'_C \sin\alpha_n + F'_T \cos\alpha_n . \tag{21}$$

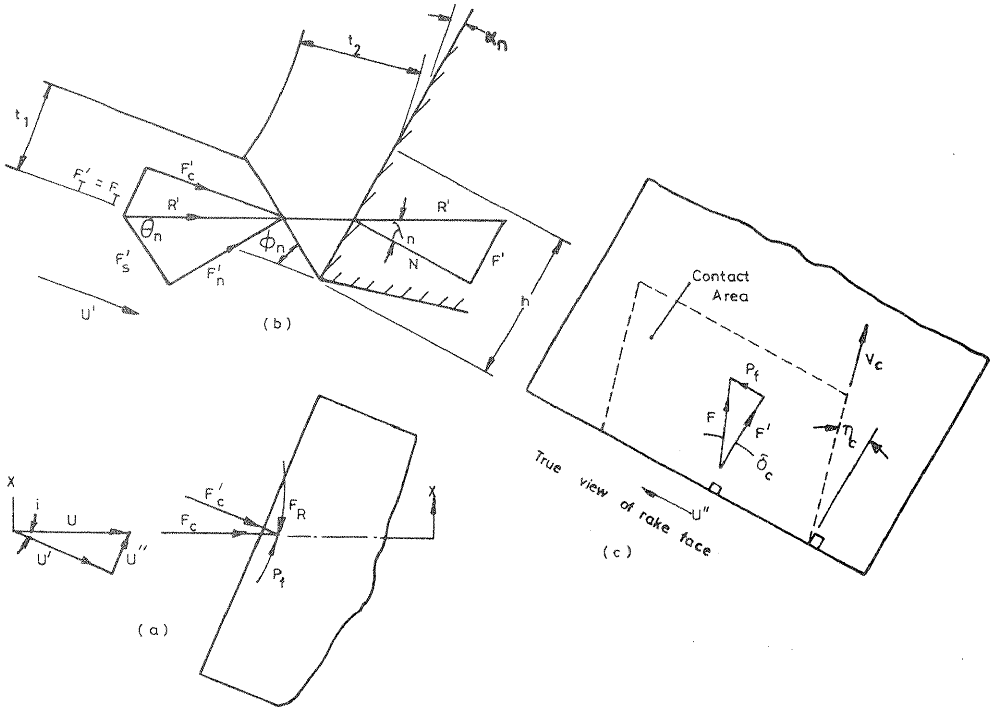


FIG. 5. Geometrical relationship in oblique machining.

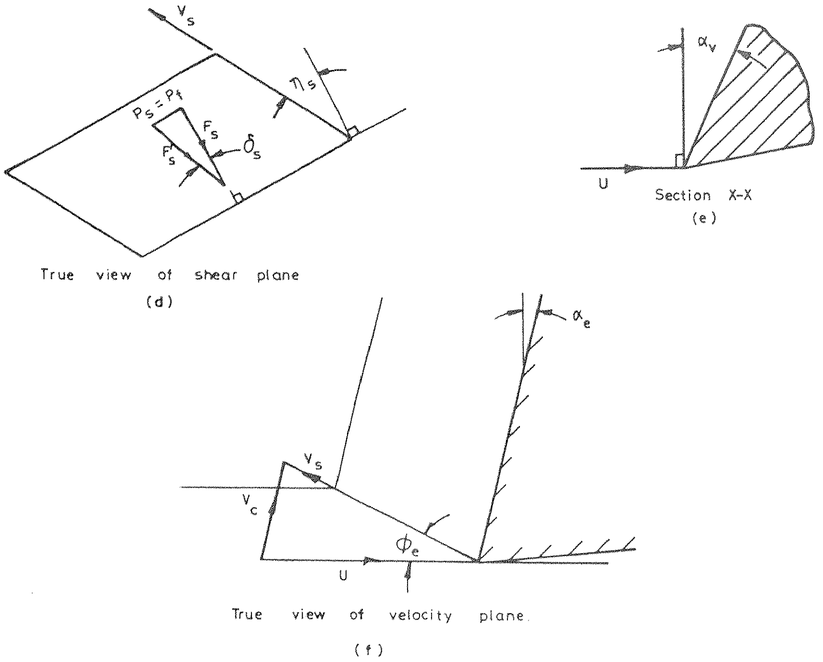


FIG. 5. (Contd.).

The shear stress at the tool chip interface due to F' is given by

$$\tau'_{int} = \frac{F'}{hw'} \tag{22}$$

where w' is the equivalent width of cut for step (i) and h the tool–chip contact length. It can be shown [9] that for the present work h can be estimated from the equation

$$h = 1.82\, t_1 \frac{\sin\theta_n}{\sin\phi_n \cos\lambda_n}. \tag{23}$$

The equivalent stress is

$$\sigma'_{int} = \sqrt{3}\,\tau'_{int}. \tag{24}$$

To find the actual value of ϕ_n, the chip strength at the interface is considered as a function of the velocity – modified temperature which, as previously described, is a combined effect of temperature and strain-rate. The chip strength, k_{chip}, curve shown in Fig. 4 can be represented by the equations

$$\begin{aligned} k_{chip} &= 33450 - 6250\sin\left(\frac{T_{mod} - 410}{165}\cdot\frac{\pi}{2}\right) \\ &\quad \text{for } 410 < T_{mod} \leqq 575 \\ k_{chip} &= 29450 + 2250\sin\left(\frac{T_{mod} - 702}{265}\cdot\pi\right) \\ &\quad \text{for } 575 < T_{mod} < 900. \end{aligned} \tag{25}$$

The equivalent chip stress can be written as

$$\sigma_{chip} = \sqrt{3}\,k_{chip} \tag{26}$$

T_{mod} (°K) in equation (25) is the velocity-modified temperature which considering the normal plane is given by equation

$$T_{mod} = T_{int}\left(1 - v\log\frac{\dot{\gamma}'_{int}}{\dot{\gamma}_0}\right). \tag{27}$$

Here v and $\dot{\gamma}_0$ are constants as defined before. $\dot{\gamma}'_{int}$ is given by equation (20) (with $V = V'$) and T_{int} is calculated from equation (16), which can be written as:

$$T_{int} = T_w + T'_{SZ} + \frac{T'_M}{2}. \tag{28}$$

In this expression T_w is the initial work temperature (taken as 20°C), T'_{SZ} is the temperature rise in the shear zone and T'_M is the maximum temperature rise along the tool–chip interface. T'_{SZ} is still assumed to be given by equation (13) with F'_C, F' and w' replacing F_C, F, and w, that is

$$T'_{SZ} = \frac{1-\beta}{12\,J\rho S t_1 w'}\left[F'_C - \frac{F'\sin\phi_n}{\cos(\phi_n - \alpha_n)}\right]. \tag{29}$$

This implies that the influence of P_s is neglected.

T'_M can be found from equation (17) with τ'_{int}, U', ϕ_n and α_n replacing τ_{int}, U, ϕ, and α, respectively. The influence of P_f is neglected.

For given values of cutting conditions (that is α_n, t_1, U, w and i) the normal shear angle ϕ_n and corresponding cutting forces, F_C, F_T, and F_R can be found as follows.

A value of ϕ_n is taken and used with equation (2) (with V'_s replacing V_s) to find $\dot{\gamma}_{AB}$. The corresponding value of n is then found from equation (11) and substituted in equation (3) to give θ_n which is used with equation (5) to give λ_n. The corresponding values of F'_C and F'_T can be found from the geometry of Fig. 5, that is

$$F'_C = \frac{t_1 w' k_{AB}\cos(\lambda_n - \alpha_n)}{\sin\phi_n\cos(\phi_n + \lambda_n - \alpha_n)} \tag{30}$$

and

$$F'_T = \frac{t_1 w' k_{AB}\sin(\lambda_n - \alpha_n)}{\sin\phi_n\cos(\phi_n + \lambda_n - \alpha_n)} \tag{31}$$

where k_{AB} is given by equation (10) for the corresponding strain-rate. Values of F'_C and F'_T are substituted in equation (21) to give F' which is then used with equations (22), (23) and (24) to obtain σ'_{int}. Equations (27), (28) and (29) are used to calculate T_{mod} which is then used with equations (25) and (26) to give σ_{chip}.

Typical curves of σ'_{int} and σ_{chip} against ϕ_n obtained in this way are given in Fig. 6. The intercept of the two curves is taken as the solution of ϕ_n since at this point the resolved stress σ'_{int} is equal to the value of σ_{chip} given by the corresponding temperature and strain-rate and the process is in equilibrium.

The corresponding cutting force components F_C, F_T and F_R can be found from the geometry shown in Fig. 5, that is

$$\begin{aligned} F_C &= F'_C\cos i + P_f\sin i \\ F_T &= F'_T \\ F_R &= F'_C\sin i - P_f\cos i \end{aligned} \tag{32}$$

where $P_f = F'\tan\eta_c$

$$= [F'^2_C + F'^2_T]^{1/2}\sin\lambda_n\tan\eta_c \tag{33}$$

where η_c has been given by Stabler [10] in the equation

$$\tan(\phi_n + \lambda_n) = \frac{\tan i\cos\alpha_n}{\tan\eta_c - \sin\alpha_n\tan i} \tag{34}$$

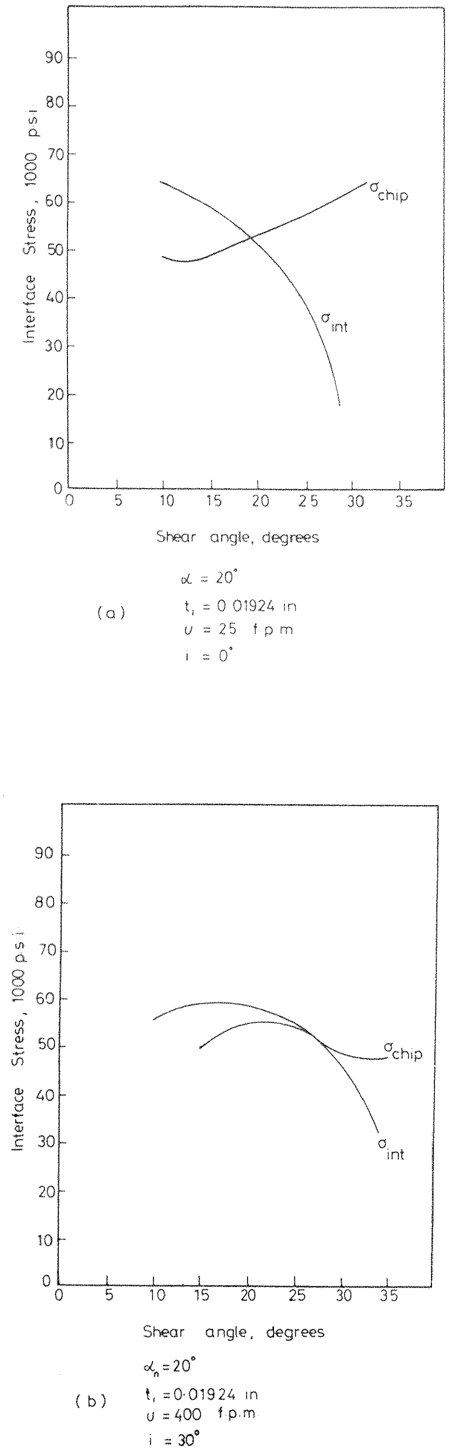


FIG. 6. Curves of tool–chip interface stress showing how solution for shear angle is obtained.

which can be rearranged as

$$\tan \eta_c = \frac{\tan i \cos \alpha_n}{\tan(\phi_n + \lambda_n)} + \sin \alpha_n \tan i. \tag{35}$$

Thus, for given cutting conditions with the appropriate material properties known, it is now possible to predict ϕ_n, η_c and the three cutting force components F_C, F_T and F_R.

A computer program has been written for performing the above calculations, including plotting of the σ'_{int} and σ_{chip} curves for determining the solution point and the corresponding cutting force components and chip flow angle.

Values of ϕ_n, η_c and cutting forces found in the above described method are given in Figs. 7, 8 and 9. These include the predicted results for $i = 0$, i.e. orthogonal conditions. The corresponding experimental results are also given.

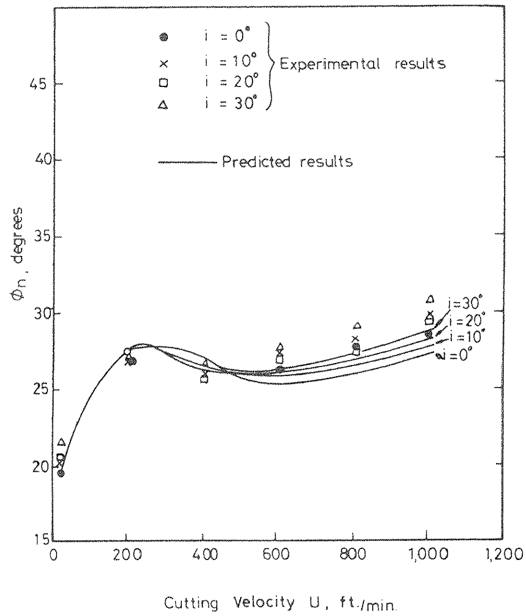


FIG. 7. Predicted and experimental shear angle.

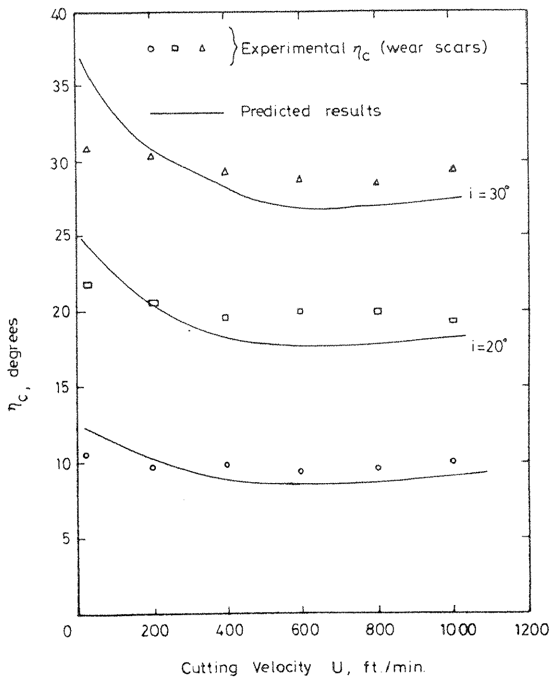


FIG. 8. Predicted and experimental chip-flow angles.

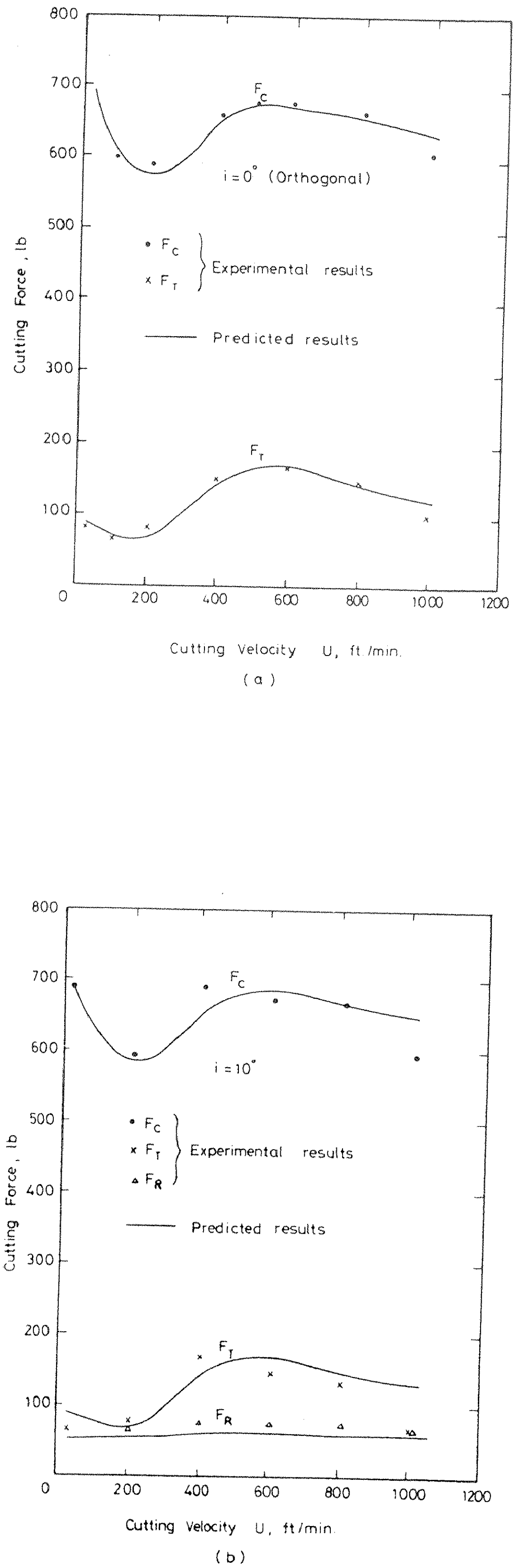
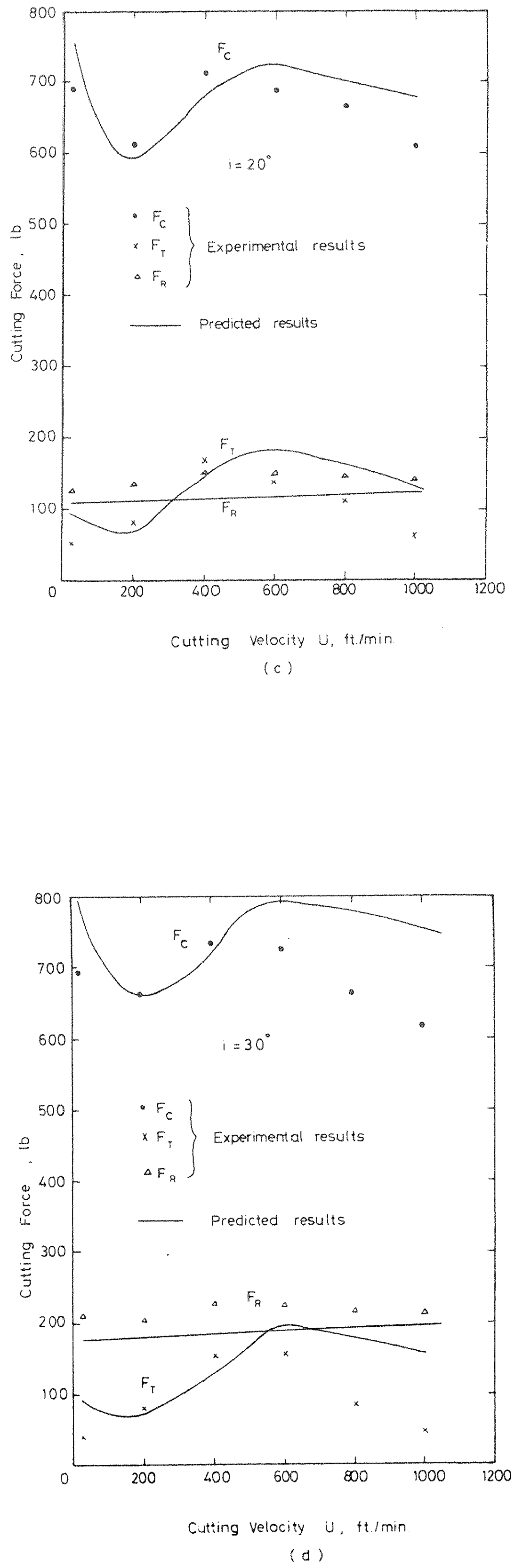


FIG. 9. Predicted and experimental cutting forces.



Cutting Force , lb
Cutting Velocity U, ft./min
F_C
F_T
F_R
i = 20°
• F_C × F_T △ F_R Experimental results
Predicted results
(c)
i = 30°
(d)

FIG. 9. (Contd.).

DISCUSSION

The agreement between the predicted and experimental values of shear angle (Fig. 7) and chip-flow direction (Fig. 8) are good. The theory also can be seen to predict successfully the main trends in cutting forces with changes in inclination angle and cutting speed (Fig. 9). The predicted results also show, in agreement with experiment, that as the inclination angle i is increased, the third component of cutting force F_R increases although its value is somewhat underestimated. At 30° inclination angle, the theory overestimates the other two cutting force components, F_C and F_T, when the cutting velocity is above 600 ft/min (Fig. 9). However the theory shows excellent agreement with experimental values of F_C and F_T at lower inclination angles (which are used in most practical applications) for all cutting velocities considered.

The chip flow direction predicted by the present theory is in good agreement with the previous work [1]. For values of cutting velocity below 500 ft/min the shear angle and cutting force predicted by both approaches are also in good agreement. However, as cutting velocity further increases, the present theory underestimates the values of shear angle and this is consistent with cutting forces being overestimated.

The mean friction angle λ, which was used in the previous work [1] to describe the frictional condition at the tool–chip interface has been replaced by the shear strength of the chip material at the tool–chip interface. This shear strength has been shown (Fig. 4) to vary in a predictable way with the velocity-modified temperature at the interface and therefore to be far more useful as a measure of friction. Thus the predictive value of the oblique machining theory is greatly enhanced. The work is being extended to tools with side cutting edge angle, side rake angle and a wider range of α_n.

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