High-Speed Machining Processes: Dynamics on Multiple Scales

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Introduction

Machining operations comprise a substantial portion of the world's manufacturing infrastructure. At the 1st *CIRP International Workshop on Modeling of Machining Operations, Atlanta GA 19 May 1998,* one of the greatest contributors the scientific study of machining operations, Dr. Eugene Merchant, estimated that 15% of the value of all mechanical components manufactured worldwide is derived from machining operations [1]. Other studies have found that domestic expenditures on machining amount to between 3% and 10% of the annual U.S. gross domestic product (GDP): between $240 to $850 billion dollars for 1998 [2,3]. However, despite its obvious economic and technical importance, machining remains one of the least understood of manufacturing operations, and machining parameters are still chosen primarily through empirical testing and the experience of machine operators and programmers. This approach is costly, and while databases have been developed from large numbers of empirical tests [4,5], these databases lose relevance as new tool materials, machines and workpiece materials are developed.

These difficulties have been accentuated by very rapid changes in machining technology during the past decade. Chief among these changes has been the rapid successful commercialization of reliable *high-speed machining systems*. The components that have enabled the development of high-speed machining include: (1) spindles capable of speeds exceeding 40 thousand revolutions per minute while simultaneously delivering tens of kilowatts of power to the cutting zone; (2) rigid, low-mass machine-tool structures; (3) high-speed linear slide-ways capable of coordinated linear motions at tangential speeds of up to 0.6 meters per second and accelerations of 20 meters per second squared. Machines that are designed to take advantage of these components are capable of metal removal rates that are in excess of ten times those of their conventional counterparts.

The most dramatic applications of high-speed machining have been in the manufacture of aluminum components where volumetric material removal rates can be extremely high: often thousands of cubic centimeters per minute. In the aerospace industry, high-speed machining is changing the way aircraft are manufactured by enabling the replacement of sheet-metal assemblies with machined monolithic components resulting in substantial cost savings and improved performance. A number of other benefits have been associated with the high-speed machining of aluminum including: (1) shorter machining time; (2) improved surface finish; (3) reduced thermal and mechanical stresses on the workpiece and tool; and (4) improved dynamic stability. Halley et al. [6] detail the development of high-speed machining at Boeing through cooperative research efforts with Tlusty, Smith and co-workers. This paper cites compelling examples of applications at Boeing during the past ten years. However, while recent advances in the high-speed machining of Nickel Aluminum Bronze, Titanium Alloys and nickel superalloys have been reported, progress in these more difficult-to-machine materials has been slow. The effect of material on attainable cutting speeds (circa 1992) is demonstrated in Figure 1 (after Shultz and Moriwaki [7]). Clearly the definition of "*high-speed*" in the term high-speed-machining is material dependent.

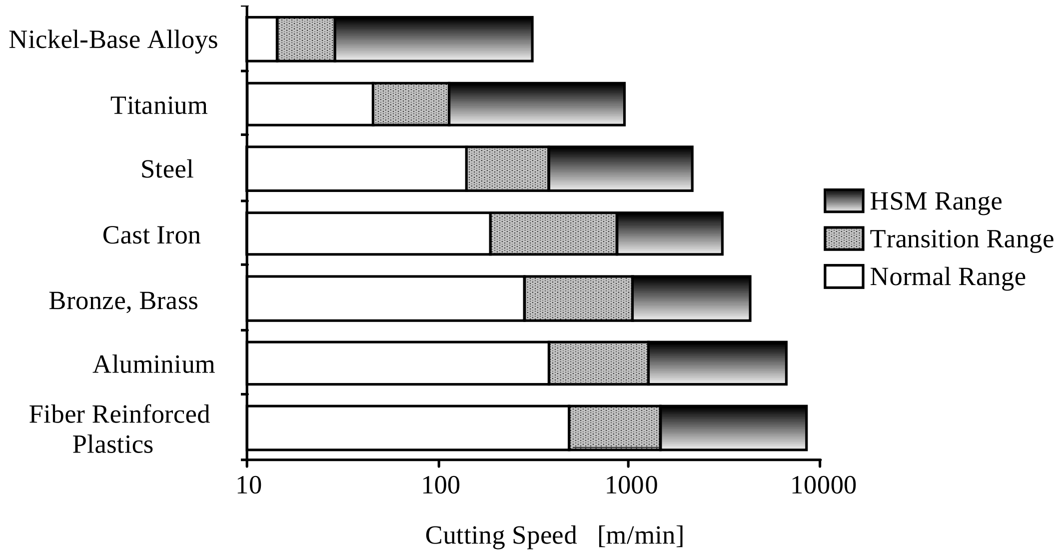

Figure 1: Attainable surface speeds in the machining of various materials (After Shultz and Moriwaki [7]).

The progress of high-speed machining in more difficult-to-machine materials has been limited by both chatter and tool wear. In high-speed machining, these factors are even more important to consider. High temperatures and material flow rates can lead to rapid catastrophic tool failure, and the solutions to dynamic stability problems may be counterintuitive (Tlusty et al. [8], Smith et al. [9] and Davies et al. [10]). In addition, past machining knowledge that has been collected in empirical databases such as the *Machining Data Handbook* [5] are out-of-date. These factors have led some manufacturers to begin seeking a more scientific approach to the problem (e.g. the Assessment of Machining Models Effort [11] among NIST, Ford General Motors and Caterpillar). There is a drive to use sophisticated finite-element and dynamic simulation software to reduce the need to generate and maintain costly empirical databases that keep pace with rapidly changing technology. Machining involves a complex interaction among dynamic phenomena occurring on wide range of different length and time scales. For example tool wear rates and the forces that may produce vibrations of the entire machine tool structure (meters in dimension) are primarily governed by the thermal, stress and strain-rate conditions in a region at the tool-chip interface. This region has characteristic dimensions that may be as small as a few micrometers. A related observation can be made about the operative time-scales of the interacting processes. This interaction between large and small-scale nonlinear phenomena makes accurate predictive modeling of high-speed machining extremely difficult. In addition, the conditions encountered in most machining operations are so extreme that there is pressing need for adequate data on the behavior of a material under conditions of high-strain, strain-rate and temperature (Childs [12]).

Survey of the Field of Machining Research

The concept of high-speed machining originated in Germany with the work of Salomon in the late 1920s [13]. Salomon conducted machining experiments in which the temperatures at the tool-chip interface were measured as a function of cutting speed for a number of different materials. These measurements showed that, while there was an initial increase in temperature with cutting speed in all materials, this trend always reversed as speeds were increased beyond a certain critical speed that was related to the material being cut. While he did not explain his results, and the results have not proved reproducible, Salomon's work was the first to suggest that counterintuitive phenomena may result from the complex nonlinear character of the plastic flow that exists at the tool-chip interface. This work led Salomon and others to postulate the existence of a high-speed machining regime that has sparked a number of efforts, detailed by King [13]. These efforts have contributed to the rapid commercialization of high-speed machining that has occurred over the last ten years.

Salomon's work was not the first scientific study of machining to be conducted. In fact such studies date back more than a century to the early work of Von Mises, Mallock [14] and Taylor [15] in the middle and late 1800s. Many of these efforts have focused on the fundamental mechanics of the plastic flow that is generated at the tool-chip interface. The next period of rapid development after Salomon occurred in the 1930's and 1940's. The study of machining mechanics was for the first time placed on a solid physical and mathematical foundation by the work of Piispanen [16] and Ernst and Merchant [17,18,19]. Progress continued in the studies of Merchant [20,21], Field and Merchant[22], Drucker [23], Shaw and Finnie [24], Lee [25] and Rice [26]. More advanced ideas from plasticity, thermo-plasticity, and materials science have also been introduced to analyze various chip-formation phenomena including: (1) continuous chip formation (Lee and Shaffer [27], Cook, Finnie and Shaw [28], Hill [29], Roth[30], Oxley [31], Ramalingham and Black [32], and Von Turkovich[33]); (2) built-up edge (Ernst and Martelotti [34]); (3) shear localization (Recht [35], Shaw [36], Komanduri [37], Molinari et al. [38], Davies and Burns [39]); (4) periodic fracture (Shaw and Vyas [40]); and segmental (Rice [26], Komanduri and Brown [41]). More recently, numerical studies have become a powerful new tool for understanding machining and have developed from fairly basic finite-element simulations (Strenkowski and Athavale [42] Childs[12]) into more sophisticated commercially available simulation codes that have drawn from ideas in armor penetration problems (Marusich and Ortiz [43]) and forming (Cerretti et al. [44]). Other models seek to treat the force problem separately and rely on empirical measurements of the *specific cutting energy* [45-48]. Still others seek to treat the thermal problem separately[49-53].

Models aimed at describing the vibratory motions of the machine-tool structure have developed in parallel to models of steady-state chip formation, which assume a rigid tool. These models utilize ideas from models of chip formation to develop approximate expressions for the cutting forces that result in such phenomena as: (1) driven vibration; (2) mode-coupling; (3) stick-slip oscillation; and (4) regeneration. However, vibration models generally rely heavily on empirical determination of *cutting force coefficients* that provide a fairly crude representation of the chip formation process. Most of these models have focused on the phenomenon of regenerative chatter, which is arguably the most detrimental type of machine tool vibration. Arnold [54] first suggested regeneration of waves on the workpiece as a potential cause of chatter, but did not fully recognize its importance. Tobias and Fishwick [55], Tlusty and Polacek [56,57] and Merrit placed the problem in a more mathematical framework and suggested that stability information could be compactly represented in the form of stability charts. Following these pioneering efforts, there have been many efforts to understand regenerative stability in machining operations based on the theory of *delay-differential equations*. Notable experimental and analytical efforts include Tobias [58], Hanna and Tobias [59], Tlusty [60], Stepan [61], Shridar et al. [62,63], Minis and Yanushevsky [64], and Altintas and Budak [65]). Other efforts have focused on more subtle aspects of the chatter problem including: (1) process damping (Tlusty [60]); (2) time-varying terms in milling (Shridar et al. [62,63], Minis and Yanushevsky [64] and Altintas and Budak [65]); and (3) nonlinearity in turning (Nayfeh et al. [66], Pratt et al. [67], Johnson and Moon [68], Kalmar-Nagy et al. [69] and Gilsinn et al. [70]) and milling (Hanna and Tobias [59], Zhao et al. [71]). While turning and milling problems have provided the major focus for studies of regenerative phenomena, some efforts have focused on chatter phenomena in other machining operations such as drilling and reaming [72], and the dual delay problem in cylindrical grinding [73,74,75] and specifically high-speed machining [76,77]. Less research has been focused on vibrations that do not develop as a result of regeneration. Notably the phenomenon of tool whirl or mode coupling has been examined by Tlusty [60] and the phenomenon of stick-slip oscillations coupled to the formation of segmental chip has been examined by Komanduri and Brown [37]. This paper shows experimental results that suggest supercritical instability in this problem. Grabec [78,79] has treated this problem from the perspective of nonlinear dynamics showing that complex unstable motions may occur when the cutting forces are a nonlinear function of cutting speed and more recently utilized ideas from nonlinear dynamics to device chatter detection methods [80].

The wide range of models, resulting from differing levels of empiricism, makes it extremely difficult to compare the results of different models. This is often daunting to those who most need to use the models in practice. This situation is probably common to other manufacturing operations that involve the interaction of diverse physical phenomena. Critical aspects of the high-speed machining problem that have not been addressed adequately are: (1) tool wear and its dependence on the material flow in difficult-to-machine materials; (2) transient regenerative vibrations that result from continuously varying cutting parameters as encountered in contour-machining operations; (3) the effect of the intermittent cut in milling on stability [81] and tool wear rates; (4) mechanics-based models for non-continuous chip formation; (5) the modeling and design of modern machine-tool structures, actuators and drive systems.

The Dynamics of High-speed Machining

Like all machine-tools, high-speed machines must accurately position a cutting tool relative to a workpiece, often following a complex path that may require up to six independent actuator motions to produce. Typically, the tools or workpiece are axi-symmetric and thus one actuator is the machine spindle which produces rotation of the tool or workpiece about an axis that is arbitrarily positioned in space by the actions of the other five actuators.

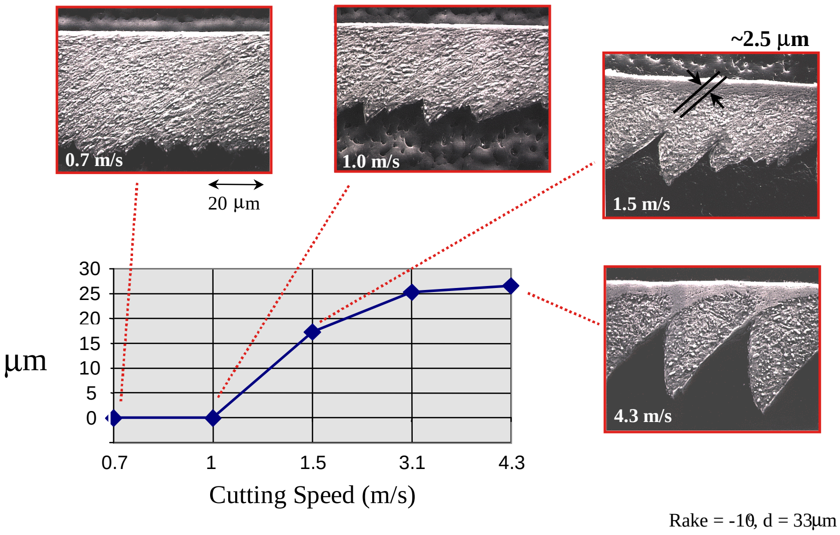

Figure 2: Variation in mean segment spacing as a function of cutting speed.

For high-speed machining, each of the actuators must be capable of producing large forces/torques at elevated speeds (and therefore high power output) without compromising the accuracy of the motions. The spindle typically generates most of the power needed to remove material. The accuracy of the finished product is therefore dependent on a number of factors: (1) the accuracy of the motions produced by the machine tool during operation; (2) the nature of the plastic flow generated at the tool chip interface; (3) the effect of the cutting operation on the motions of the machine and workpiece; and (4) the various ancillary functions such as coolant flow and removal of chips from the workzone. At high-speeds many of the phenomena associated with machining change character. For example regenerative chatter becomes so important that productive high-speed machining is difficult without at least an intuitive understanding of its behavior. Below we discuss each of the aspects of high-speed machining, focusing specifically on areas where improved understanding of the dynamics would likely lead to more rapid progress.

Tool-chip Interface

The chip formation process in machining involves a very high-strain-rate, high-temperature nonlinear plastic flow of material. This plastic flow generates the local stresses on the cutting tool, the temperature distribution at the tool chip-interface, and determines the condition of the workpiece material after the chip has been removed. These local quantities determine the global forces on the machine-tool structure, which in turn result in its dynamic and static deformations. Furthermore they determine the rates of various physical phenomena that drive tool wear, such as chemical reactions, abrasive wear, and diffusion. They also determine the power that must be generated to effect the removal of material and thus influence the amount of heating produced by the various driving components of the system. This heating results in thermally induced strains in the structure that compromise accuracy. Thus, in order to predict adequately the behavior of a machining system on a global level, accurate information about the plastic flow in the immediate vicinity of the tool is critical. It is the opinion of the authors that these flows cannot be treated as quasistatic. Their behavior must ultimately be described in the context of the stability of dynamic systems.

It has been demonstrated through careful observation that this plastic flow can exhibit at least five distinct dynamic flow patterns that are manifested in the following chip forms: (1) discontinuous; (2) continuous; (3) continuous with built-up edge; (4) shear localized; and (5) segmental (continuous with periodic variation in thickness). A unified approach to the problem would describe each type of chip formation as a stable dynamic equilibrium of the partial differential equations describing the elastic-plastic flow. For high-speed machining the two types of chip formation that are of the most importance are continuous and shear localized. In high-speed machining of a majority of materials a transition from continuous to shear localized chip formation will occur at some critical speed [35]. Figure 2 shows the transition from steady state to shear-banded (oscillatory) flow in hardened steel at a cutting speed between 1.0 m/s and 1.5 m/s. We have demonstrated through simplified models that this transition is the result of a *Hopf bifurcation* in the dynamic system describing the nonlinear material flow.

High-speed machining research would benefit from improved models and descriptions of inhomogeneous chip formation processes including an adequate description of the shear flow along the rake face of the tool. The effect of these flows on the chemical, diffusive and mechanical phenomena (such as fatigue) that produce tool wear also need to be addressed in order to assess the effect of increased cutting speed on tool wear rates. In addition, the effect of the material flow on the vibrations of the machine-tool structure has not been directly addressed. Komanduri and Brown [37] demonstrated the coupling of machine-tool vibrations and segmented chip formation which they attributed to stick slip oscillation. However, to our knowledge, the coupling between oscillatory material flow and the machine tool-structure has not been adequately modeled. At the *1st CIRP International Workshop on Modeling of Machining Operations* [1], it was suggested that one possible contribution of finite-element models would be the generation of force databases for chatter analysis and simulations. Verification of the validity of these models also requires improved measurements of the stresses, temperatures, strain-rates and strains that develop in machining.

Machine-tool Structure & Control System

A number of recent modifications to the design of machine tools may have a substantial effect os the development of high-speed machining systems and on the dynamics and on the ease of modeling these systems.

Traditional machining systems have consisted of serially-coupled, stacked components, each capable of linear or rotary motion with one degree-of-freedom. Typically linear axes have been driven by rotary actuators coupled by high-force-gain lead-screws, ball-screws and rack-and-pinion drives. Research on the design of such drive systems has enabled their use in high-speed machining centers, producing feed rates of up to 1 m/s with accelerations of 1 g.

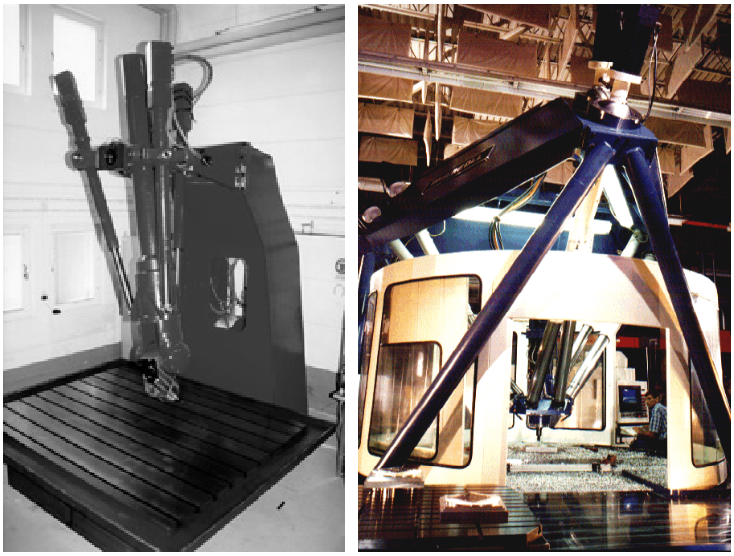

Figure 3: Two examples of parallel machine-tool

For a small work volume, the acceleration of the machine tool axes is more important than the maximum speed in determining the ultimate time it takes to manufacture a component. To address this issue, some machine-tool manufacturers have begun to explore *linear motors* as an alternative drive system for high-speed machines. These systems are capable of more than twice the maximum speeds and accelerations of mechanical drives. However, they also have a number of disadvantages: (1) low force amplification factors make them more sensitive to cutting forces and changes in the inertia of the machine-tool structure that may occur as axis orientations change and workpiece mass is added or removed; (2) because they require strong permanent magnets, ferrous machining chips may collect on the motor housings; (3) they generate a large amount of heat, thereby making a substantial (and often dominant) contribution to the thermal errors of the system. However, improvements in accelerations and speeds are so attractive that they motivate manufactures to attempt to find solutions to these problems rather than abandon the technology.

Another innovative design idea for high-speed machines is the development of parallel architecture machine-tool structures. From the perspective of high-speed machining, the development of the new machines addresses a major disadvantage of serial construction: the necessity for some axes to carry the additional mass with other axes and their actuators. Parallel machines are made possible by the development of powerful CNC controllers able to account for geometric complexity in the controller software. These systems can use traditional ballscrew drives or linear motor systems as actuators. Two examples are shown in Figure 3. It has been claimed that this type of machine can be stiffer, have lower mass and higher accelerations, and be more accurate than conventional designs [82,83]. However, these advantages have not yet been realized in practice due to a number of disadvantages including: (1) parasitic bending of the struts due to imperfect joints; (2) magnification of thermally induced errors due to strut length; (3) difficulty in obtaining direct position feedback along the struts and in determining the locations of the axes of rotation for each joint; (4) variation in system kinematics, statics, and dynamics within the work volume. In addition, Tlusty [84] has recently argued that many of the advantages of these parallel machines may be overstated and are not practically realizable. Despite these formidable challenges, much work on parallel machines continues with the final verdict on comparison with conventional construction as yet unknown. Certainly, the new designs are competing with years (perhaps centuries) of experience using conventional constructions, and therefore, prediction of their ultimate performance is difficult.

Design innovations associated with high-speed machines have a substantial effect on the ability of engineers to model their dynamic performance. Such dynamic models are now of great practical importance, since the feed motions of high-speed machines cannot be modeled as quasi-static, and system vibrations, particularly as pertaining to regenerative chatter, are of tremendous importance. This has led machine-tool manufacturers to begin addressing: (1) the repeatability of machine-tool dynamics from machine to machine; (2) the development of dynamic models of machine-tools that may be marketed and sold for use in tuning for regenerative chatter. These concerns bring out a number of fundamental dynamics issues that include: (1) modeling of multi-body dynamic systems with potentially nonlinear mechanical connections; (2) control of multi-body nonlinear systems with unprecedented speed and accuracy; (3) measurement of system dynamics in the presence of closely spaced, well-damped modes; (4) coupling of machine-tool dynamic models to accurate regenerative chatter models and prediction of performance in real-time. For regenerative chatter, only the dynamic response at the tool-tip is needed. Therefore, methods such as *receptance coupling* can be used to determine the necessary dynamic behavior, if the system components are approximately linear [85]. In receptance coupling, frequency response measurements of the individual components of an assembly (i.e. spindle, tool holder, and tool) are combined to predict the tool-point dynamic response. Rather than requiring a separate measurement for each spindle/holder/tool combination, any assembly can be predicted from the component data and information about the dynamic characteristics of the connections. In high-speed machining where *tool-tuning* [8-10] will likely become a necessary practice, this type of component analysis is particularly useful for predicting the effects of changes in tool geometry on the tool-tip response behavior. For control systems and particularly for design this is not the case. The problem of nonlinearities in mechanical couplings is an old one but is particularly relevant to machine-tool systems. In this respect linear motors are generally more easily modeled giving them an additional advantage over mechanical drive systems. This emphasizes a general concept that has not received much attention in the machine tool community: *design for (dynamic) "modelability"*.

Stability

In the opinion of the authors, traditional regenerative chatter stability theory is adequate for providing a practically usable characterization of the stability of most simple cutting operations that can be approximated with an orthogonal cut. The primary limitations of these models do not lie in the basic regenerative model, but rather in the adequate (linear and nonlinear) characterization of the cutting process (e.g. the force dependence on the state variables) and the machine-tool structure dynamics. These issues are addressed in the sections above. Where analytical regenerative chatter theory fails to provide accurate prediction of the system stability is in: (1) highly interrupted cutting; (2) cutting with complex tool geometry; and (3) contour machining where the system parameters change rapidly with time. These will now be discussed n more detail.

Low Immersion Machining

Traditional regenerative stability theory predicts sets of spindle speeds that are most resistant to the development of chatter. Considering regeneration alone, these spindle speeds (tooth engagement frequencies in milling) are approximately at integer fractions of the natural frequencies of the most flexible modes of the machine-tool structural loop. However, for highly interrupted machining processes, where the ratio of time spent cutting to not cutting (ρ) is small, the assumptions of the traditional theory break down. We have proposed a new stability theory for interrupted machining that predicts a doubling in the number of optimally stable speeds as the value of ρ is reduced. This will be discussed in more detail in the technical sessions of the workshop.

Contour Machining with Complex Tool Geometry

Many high-speed machining operations involve the production of contoured surfaces using ball end mills. To generate the necessary fine surface finish, these operations involve numerous light cuts made at very high speeds. The machining parameters vary continuously during each cut. The effect of complex tool geometry and rapid changes in machining parameters on the process stability has only been addressed by complex numerical simulations. Accurate analytical predictions would be extremely helpful in designing machining operations for contoured surfaces.

Concluding Remarks

The benefits of high-speed machining are driving rapid changes in the machining technology that is available for shop floor use. However, these rapid changes have quickly rendered much of the empirical knowledge that has been built up about machining over the past century invalid, placing stress on the producers and end users of this technology to generate new data appropriate for high-speed machining. The process of generating empirical data is extremely expensive, and many industrial designers and users are considering the use of simulations technology to augment the data generation process. Dynamics research can aid this development in the following areas:

- analysis of the *material flow dynamics*, particularly the effect of cutting speeds on the stress and temperature conditions at the tool-chip interface;
- *multi-body dynamical analysis* of the machine-tool structure including the dynamical properties of interfaces between components;
- research on the design of machine-tool structures for *dynamic repeatability*;
- analysis of the dynamics of *parallel machine-tool structures*;
- dynamic modeling and control of *high-speed axis drive systems*;

- development of approximate analytical solutions for the stability of complex contour machining and nonlinear models of interrupted machining .

Each of these areas will be discussed in the presentation.

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