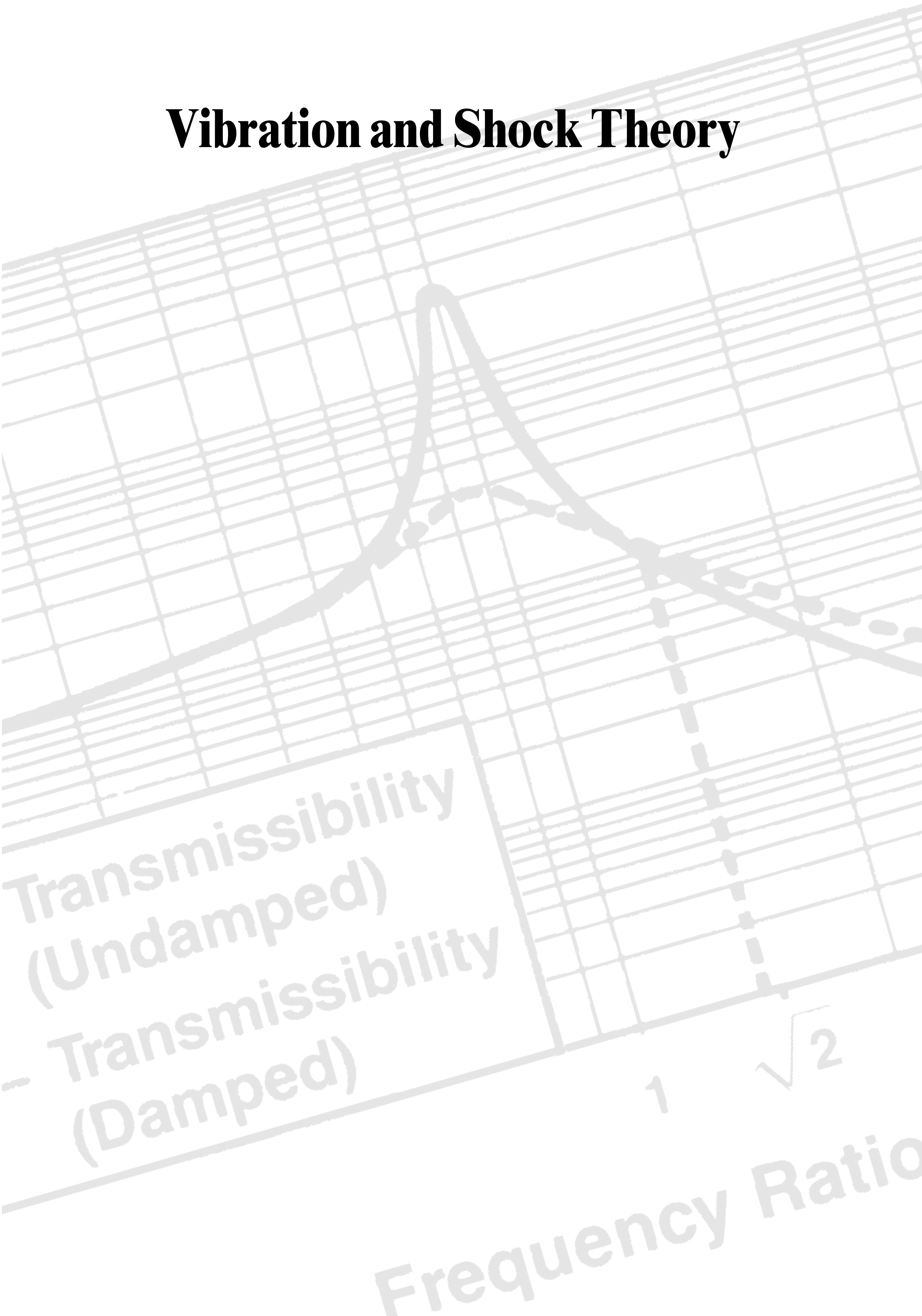
Vibration and Shock Theory



Introduction

This catalog has been prepared to assist in the selection of Lord products to solve a variety of vibration and shock isolation problems in aerospace equipment. The theory applies to any problem in the field of vibration and shock isolation and many of the products shown in this catalog may be used for applications other than the protection of electronic equipment.

Before attempting to apply any isolator, it is important to know as much as possible about the conditions under which it will be used and the sensitivity (fragility) of the equipment to be mounted. This knowledge must be coupled with an understanding of the various types of vibration and shock isolators which might be applied to a given problem. Depending on the type of isolator, the material from which it is made and the operating conditions, the performance of the isolator and its effectiveness can vary widely. These factors must be considered, and the proper accommodations made to theory, to arrive at a reasonably accurate estimate of the performance of the isolated system.

The following discussion presents the basic theory and some trends of material performance in order to address the peculiarities of the real world of vibration and shock theory.

Terms and Definitions

There are a number of terms which should be understood before entering into a discussion of vibration and shock theory. Some of these are quite basic and may be familiar to many of the users of this catalog. However, a common understanding should exist for maximum effectiveness.

Center-of-Gravity System — An equipment installation wherein the center of gravity of the equipment coincides with the elastic center of the isolation system.

Damping — The "mechanism" in an isolation system which dissipates energy. This mechanism controls resonant amplification (transmissibility).

Decibel — (db) — A dimensionless expression of the ratio of two values of some variable in a vibratory system. For example, in random vibration the ratio of the power spectral density at two frequencies is given as:

$$db = -10\text{Log}_{10}\frac{S_{f_1}}{S_{f_2}}$$

Deflection — The movement of some component due to the imposition of a force. In vibratory systems, deflection may be due to static or dynamic forces or to the combination of static and dynamic forces.

Degree-of-Freedom — The expression of the amount of freedom a system has to move within the constraints of its application. Typical vibratory systems may move in six degrees of freedom—three translational and three rotational modes (motion along three mutually perpendicular axes and about those three axes).

Dynamic Matching — The selection of isolators whose dynamic characteristics (stiffness and damping) are very close to each other for use as a set on a given piece of equipment. Such a selection process is recommended for isolators which are to be used on motion sensitive equipment such as guidance systems, radars and optical units.

Dynamic Disturbance — The dynamic forces acting on the body in a vibratory system. These forces may be the results of sinusoidal vibration, random vibration or shock, for example.

Elastomer — A generic term used to include all types of "rubber"— natural or synthetic. Many vibration isolators are manufactured using some type of elastomer. The type depends on the environment in which the isolator is to be used.

Fragility — The amount of vibration or shock which a piece of equipment can take without malfunctioning or breaking. In isolation systems, this is a statement of the amount of dynamic excitation which the isolator can transmit to the isolated equipment.

Free Deflection — The amount of space an isolated unit has in which it can move without interfering with surrounding equipment or structure. This is sometimes called "sway space."

"g" level — An expression of the vibration or shock acceleration level being imposed on a piece of equipment as a dimensionless factor times the acceleration due to gravity.

Isoelastic — A word meaning that an isolator, or isolation system, exhibits the same stiffness characteristics in all directions.

Isolation — The protection of equipment from vibration and/or shock. The degree (or percentage) of isolation necessary is a function of the fragility of the equipment.

Linear (properties) — A description of the characteristics of an isolation system which assumes that there is no variation with deflection, temperature, vibration level, etc. This is a simplifying assumption which is useful for first approximations but which must be treated carefully when dealing with critical isolation systems.

Loss Factor — A property of an elastomer which is a measure of the amount of damping in the elastomer. The higher the loss factor, the higher the damping. Loss factor is typically given the Greek symbol "η". An approximation may be made that loss factor is equal to the inverse of the resonant transmissibility of a vibratory system. The loss factor of an elastomer is sensitive to the loading and ambient conditions being imposed on the system.

Modulus — A property of elastomers (analagous to the same property of metals) which is the ratio of stress to strain in the elastomer at some loading condition. Unlike the modulus of metals, the modulus of elastomers is non-linear over a range of loading and ambient conditions. This fact makes the understanding of elastomers and their properties important in the understanding of the performance of elastomeric vibration and shock isolators.

Natural Frequency — That frequency (expressed as "Hertz" or "cycles per second") at which a structure, or combination of structures, will oscillate if disturbed by some force (usually dynamic) and allowed to come to rest without any further outside influence. Vibratory systems have a number of natural frequencies depending on the direction of the force and the physical characteristics of the isolated equipment. The relationship of the system natural frequency to the frequency of the vibration or shock determines, in part, the amount of isolation (protection) which may be attained.

Octave — A doubling of frequency. This word is used in various expressions dealing with vibration isolation.

Power Spectral Density — An expression of the level of random vibration being experienced by the equipment to be isolated. The units of power spectral density are "$\frac{g^2}{Hz}$" and the typical symbol is "S_f".

Random Vibration — Non-cyclic, non-sinusoidal vibration characterized by the excitation of a broad band of frequencies at random levels simultaneously. Typically, many applications of equipment in the field of Military Electronics are exposed to random vibration.

Resilience — The ability of a system to return to its initial position after being exposed to some external loading. More specifically, the ability of an isolator to completely return the energy imposed on it during vibration or shock. Typically, highly damped elastomers have low resilience while low-damped elastomers have good resilience.

Resonance — Another expression for natural frequency. A vibratory system is said to be operating in resonance when the frequency of the disturbance (vibration or shock) is coincident with the system natural frequency.

Resonant Dwell — A test in which the equipment is exposed to a long term vibration at its resonant frequency. This test was used as an accelerated fatigue test for sinusoidal vibration conditions. In recent times, sinusoidal testing is being replaced by random vibration testing and resonant dwell tests are becoming less common.

Returnability — The ability of a system, or isolator, to resume its original position after removal of all outside forces. This term is sometimes used interchangeably with resilience.

Roll-off Rate — The steepness of the transmissibility curve being recorded during a vibration test, after the system natural frequency has been passed. This term is also used to describe the slope of a random vibration curve. The units are typically "$\frac{db}{octave}$".

Considerations In Selecting A Vibration Isolator

In the process of deciding on a vibration isolator for a particular application, there are a number of critical pieces of information which are necessary to define the desired functionality of the isolator. Some items are more citical than others but all should be considered in order to select, or design the appropriate product.

Some of the factors which must be considered are:

Weight, size, center-of-gravity of the equipment to be isolated — Obviously, the weight of the unit will have a direct bearing on the type and size of the isolator. The size, or shape of the equipment can also affect the isolator design since this may dictate the type of attachment and the available space for the isolator. The center-of-gravity location is quite important in that isolators of different load capacities may be necessary at different points on the equipment due to weight distribution. The locations of the isolators relative to the center-of-gravity—at the base of the equipment versus in the plane of the c.g., for example—could also affect the design of the isolator.

Types of dynamic disturbances to be isolated — This is basic to the definition of the problem to be addressed by the isolator selection process. In order to make an educated selection or design of a vibration/shock isolator, this type of information must be defined as well as possible. Typically, sinusoidal and/or random vibration spectra will be defined for the application. In many installations of military electronics equipment, random vibration tests have become commonplace and primary military specifications for the testing of this type of equipment (such as MIL-STD-810) have placed heavy emphasis on random vibration, tailored to the actual application. Other equipment installations, such as in shipping containers, may still require significant amounts of sinusoidal vibration testing.

Shock tests are often required of many types of equipment. Such tests are meant to simulate those operational (e.g., carrier landing of aircraft) or handling (e.g., bench handling or drop) conditions which lead to impact loading of the equipment.

Static loadings other than supported weight — In addition to the weight and dynamic loadings which isolators must react, there are some static loads which can impact the selection of the isolator. An example of such loading is that imposed by an aircraft in a high speed turn. This maneuver loading must be reacted by the isolator and can, if severe enough, cause an increase in the isolator size. These loads are often superposed on the dynamic loads.

Allowable system response — This is another basic bit of information. In order to appropriately isolate a piece of equipment, the isolator selector must know the response side of the problem. The equipment manufacturer or user should have some knowledge of the fragility of the unit. This fragility, related to the specified dynamic loadings will allow the selection of an appropriate isolator. This may be expressed in terms of the vibration level versus frequency or the maximum shock loading which the equipment can endure without malfunctioning or breaking. If the equipment manufacturer or installer is somewhat knowledgeable about vibration/shock isolation, this allowable response may be simply specified as the allowable natural frequency and maximum transmissibility allowed during a particular test.

The specification of allowable system response should include the maximum allowable motion of the isolated equipment. This is important to the selection of an isolator since it may define some mechanical, motion limiting feature which must be incorporated into the isolator design. It is fairly common to have an incompatibility between the allowable “sway space” and the motion necessary for the isolator to perform the desired function. In order to isolate to a certain degree, it is required that a definite amount of motion be allowed. Problems in this area typically arise when isolators are not considered early enough in the process of designing the equipment or the structural location of the equipment.

Ambient environment — The environment in which the equipment is to be used is very important to the selection of an isolator. Within the topic of environment, temperature is by far the most critical item. Variations in temperature can cause variations in the performance of many typical vibration/shock isolators. Thus, it is quite important to know the temperatures to which the system will be exposed. The majority of common isolators are elastomeric. Elastomers tend to stiffen and gain damping at low temperatures and to soften and lose damping at elevated temperatures. The amounts of change depend on the type of elastomer selected for a particular installation.

Other environmental effects — from humidity, ozone, atmospheric pressure, altitude, etc. — are minimal and may be typically ignored. Some external factors that may not be thought of as environmental may impact on the selection of an isolator. Such things as fluids (oils, fuels, coolants, etc.) which may be in the area of the isolators may cause a change in the material selection or the addition of some form of protection for the isolators.

Service life — The length of time for which an isolator is expected to function effectively is another strong determining factor in the selection or design process. Vibration isolators, like other engineering structures have finite lives. Those lives depend on the loads imposed on them. The prediction of the life of a vibration/shock isolator depends on the distribution of loads over the typical operating spectrum of the equipment being isolated. Typically, the longer the desired life of the isolator, the larger that isolator must be for a given set of operating parameters. The definition of the isolator operating conditions is important to any semi-reliable prediction of life.

Specification of Isolator Selection Factors — This on-line catalog includes a questionnaire, or "Data Required" form, which is helpful in the definition of the above areas of information. If the indicated information is available, the selection of an isolator will be greatly enhanced. The theory that follows in the next section is worthless if the information to apply it is not available. If an equipment designer is attempting to select an isolator from this catalog, the job will be eased by having this information available. Likewise, if a company like Lord must be consulted for assistance in the selection or design of an isolator, then the communications and accuracy of response will be improved by having such information ready.

Theory of Vibration/Shock Isolators

The solutions to most isolator problems begin with consideration of the mounted system as a damped, single degree of freedom system. This allows simple calculations of most of the parameters necessary to decide if a standard isolator will perform satisfactorily or if a custom design is required. This approach is based on the facts that:

1. Many isolation systems involve center-of-gravity installations of the equipment. That is, the center-of-gravity of the equipment coincides with the elastic center of the isolation system. The center-of-gravity installation is often recommended since it allows performance to be predicted more accurately and it allows the isolators to be loaded in an optimum manner. Figure 1 shows some typical center-of-gravity systems.

2. Many equipment isolation systems are required to be isoelastic. That is, the system translational spring rates in all directions are the same.

3. Many pieces of equipment are relatively light in weight and support structures are relatively rigid in comparison to the stiffness of the isolators used to support and protect the equipment.

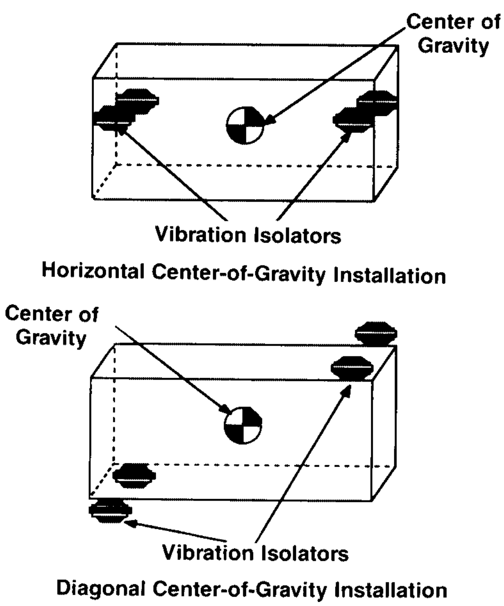


FIGURE 1
TYPICAL CENTER-OF-GRAVITY INSTALLATIONS

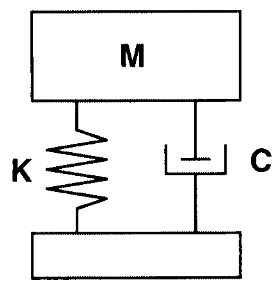
For cases which do not fit the above conditions, or where more precise analysis is required, there are computer programs available to assist the analyst. Lord computer programs for dynamic analysis are used to determine the system response to various dynamic disturbances. The loads, motions, and accelerations at various points on the isolated equipment may be found and support structure stiffnesses may be taken into account. Some of the more sophisticated programs may even accept and analyze non-linear systems. This discussion is reason to emphasize the need for the information regarding the intended application of the isolated equipment. The dynamic environment, the ambient environment and the physical characteristics of the system are all important to a proper analysis. The use of the checklist included with this catalog is recommended as an aid.

With the above background in mind, the aim of this theory section will be to use the single degree-of freedom basis for the initial selection of standard isolators. This is the first step toward the design of custom isolators and the more complex analyses of critical applications.

SINGLE DEGREE-OF-FREEDOM DYNAMIC SYSTEM

Figure 2 shows the "classical" spring, mass, damper depiction of a single degree-of-freedom dynamic system. Figure 3 and the related equations show this system as either damped or undamped. Figure 4 shows the resulting vibration response transmissibility curves for the damped and undamped systems of Figure 3.

These figures and equations are well known and serve as a useful basis for beginning the analysis of an isolation problem. However, classical vibration theory is based on one assumption that requires understanding in the application of the theory. That assumption is that the properties of the elements of the system behave in a linear, constant manner. Data to be presented later will give an indication of the factors which must be considered when applying the analysis to the real world.



M—Mass—Stores kinetic energy
K—Spring—Stores potential energy, supports load
C—Damper—Dissipates energy, cannot support load

FIGURE 2
ELEMENTS OF A VIBRATORY SYSTEM

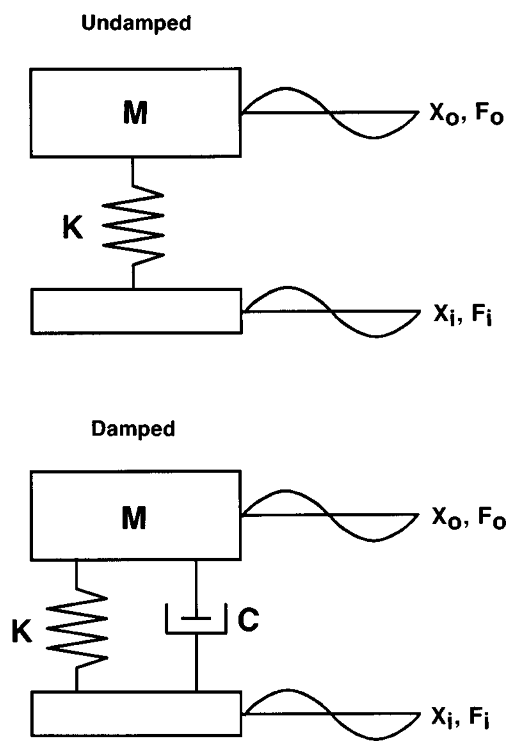


FIGURE 3
DAMPED AND UNDAMPED SINGLE DEGREE-OF-FREEDOM
BASE EXCITED VIBRATORY SYSTEMS

The equations of motion for the above model systems are familiar to many. For review purposes, they are presented here.

FOR THE UNDAMPED SYSTEM

The differential equation of motion is:

$$M\ddot{X} + KX = F(t)$$

In which it may be seen that the forces due to the dynamic input (which varies as a function of time) are balanced by the inertial force of the accelerating mass and the spring force. From the solution of this equation, comes the equation defining the natural frequency of an undamped spring-mass system:

$$f_n = \frac{1}{2\pi}\sqrt{K/M}$$

Another equation which is derived from the solution of the basic equation of motion for the undamped vibratory system is that for transmissibility—the amount of vibration transmitted to the isolated equipment through the mounting system depending on the characteristics of the system and the vibration environment.

$$T_{ABS} = \frac{1}{(1 - r^2)}$$

Wherein, "r" is the ratio of the exciting vibration frequency to the system natural frquency. That is:

$$r = \frac{f}{f_n}$$

In a similar fashion, the damped system may be analyzed. The equation of motion here must take into account the damper which is added to the system. It is:

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

The equation for the natural frequency of this system may, for normal amounts of damping, be considered the same as for the undamped system. That is,

$$f_n = \frac{1}{2\pi}\sqrt{K/M}$$

In reality, the natural frequency does vary slightly with the amount of damping in the system. The damping factor is given the symbol "ζ" and is approximately one-half the loss factor, "η," described in the definition section regarding damping in elastomers. The equation for the natural frequency of a damped system, as related to that for an undamped system, is:

$$f_{nd} = f_n\sqrt{1 - \zeta^2}$$

The damping ratio, ζ , is defined as:

$$\zeta = C/C_c$$

$$\zeta \approx \eta/2$$

Where, the “critical” damping level for a damped vibratory system is defined as:

$$C_c = 2\sqrt{KM}$$

The equation for the absolute transmissibility of a damped system is written as:

$$T_{ABS} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(2\zeta r)^2 + [(1-r^2)]^2}}$$

The equations for the transmissibilities of the undamped and damped systems are plotted in Figure 4. As may be seen, the addition of damping reduces the amount of transmitted vibration in the amplification zone, around the natural frequency of the system **(r = 1)**. It must also be noted that the addition of damping reduces the amount of protection in the isolation region **(where r > $\sqrt{2}$)**.

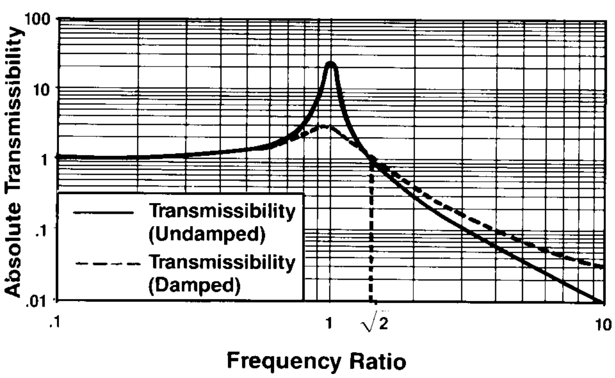


FIGURE 4
TYPICAL TRANSMISSIBILITY CURVES

In the real world of practical isolation systems, the elements are not linear and the actual system response does not follow the above analysis rigorously. Typically, elastomeric isolators are chosen for most isolation schemes. Elastomers are sensitive to the vibration level, frequency and temperature to which they are exposed. The following discussion will present information regarding these sensitivities and provide some guidance in the application of isolators for typical installations.

Elastomers for Vibration and Shock Isolation

Depending on the ambient conditions and loads, a number of elastomers may be chosen for the isolators in a given isolation system. As seen in the above discussion, the addition of damping allows more control of the system in the region of resonance. The compromise which is made here though is that isolation is sacrificed. The higher the amount of damping, the greater the compromise. In addition, typical highly damped elastomers exhibit poor returnability and greater drift than elastomers which have medium or low damping levels. The requirements of a given application must be carefully weighed in order to select the appropriate elastomer.

Within the various families of Lord products, a number of elastomers may be selected. Some brief descriptions may help to guide in their selection for a particular problem.

Natural Rubber — This elastomer is the baseline for comparison of most others. It was the first elastomer and has some desirable properties, but also has some limitations in many applications. Natural rubber has high strength, when compared to most synthetic elastomers. It has excellent fatigue properties and low to medium damping which translates into efficient vibration isolation. Typically, natural rubber is not very sensitive to vibration amplitude (strain). On the limitation side, natural rubber is restricted to a fairly narrow temperature range for its applications. Although it remains flexible at relatively low temperatures, it does stiffen significantly at temperatures below 0°F. At the high temperature end, natural rubber is often restricted to use below approximately 180°F.

Neoprene — This elastomer was originally developed as a synthetic replacement for natural rubber and has nearly the same application range. Neoprene has more sensitivity to strain and temperature than comparable natural rubber compounds.

SPE®I — This is another synthetic elastomer which has been specially compounded by Lord for use in applications requiring strength near that of natural rubber, good low temperature flexibility and medium damping. The major use of SPE I elastomer has been in vibration and shock mounts for the shipping container industry. This material has good retention of flexibility to temperatures as low as -65°F. The high temperature limit for SPE I elastomer is typically +165°F.

BTR® — This elastomer is Lord’s original “Broad Temperature Range” elastomer. It is a silicone elastomer which was developed to have high damping and a wide span of operational temperatures. This material has an application range from -65°F to +300°F. The loss factor of this material is in the range of 0.32. This elastomer has been widely used in isolators for Military Electronics equipment for many years. It does not have the high load carrying capability of natural rubber but is in the high range for materials with this broad temperature range.

BTR II® — This material is similar in use to the BTR® elastomer except that it has a slightly more limited temperature range and has less damping. BTR II may be used for most applications over a temperature range from -40°F to + 300°F. The loss factor for typical BTR II compounds is in the range of 0.18. This elastomer has better returnability, less drift, and better stability with temperature, down to -40°F. The compromise with BTR II elastomer is the lower damping. This means that the resonant transmissibility of a system using BTR II elastomeric isolators will be higher than one using BTR isolators. At the same time, the high frequency isolation will be slightly better with the BTR II. This material has found use in Military Electronics isolators as well as in isolation systems for aircraft engines and shipboard equipment.

BTR VI — This is a very highly damped elastomer. It is a silicone elastomer of the same family as the BTR elastomer described above but is specially compounded to have loss factors in the 0.60 to 0.70 range. This would result in resonant transmissibility readings below 2.0 if used in a typical isolation system. This material is not used very often in applications requiring vibration isolation. It is most often used in products which are specifically designed for damping, such as lead-lag dampers for helicopter rotors. If used for a vibration isolator, BTR VI will provide excellent control of resonance but will not provide the degree of high frequency isolation that other elastomers will provide. The compromises here are that this material is quite strain and temperature sensitive, when compared to BTR and other typical Miltronics elastomers, and that it tends to have higher drift than the other materials.

"MEM" — This is an elastomer which has slightly less damping than Lord's BTR® silicone, but which also has less temperature and strain sensitivity. The typical loss factor for the MEM series of silicones is 0.29, which translates into a typical resonant transmissibility of 3.6 at room temperature and moderate strain across the elastomer. This material was developed by Lord at a time when some electronic guidance systems began to require improved performance stability of isolation systems across a broad temperature range, down to -70°F, while maintaining a reasonable level of damping to control resonant response.

"MEA" — With miniaturization of electronic instrumentation, equipment became slightly more rugged and could withstand somewhat higher levels of vibration, but still required more constant isolator performance over a wide temperature range. These industry trends led to the development of Lord MEA silicone. As may be seen in the material property graphs of Figures 5 through 8, this elastomer family offers significant improvement in strain and temperature sensitivity over the BTR® and MEM series. The compromise with the MEA silicone material is that it has less damping than the previous series. This results in typical loss factors in the range of 0.23 - Resonant Transmissibility of approximately 5.0. The MEA silicone also shows less drift than the standard BTR series elastomer.

"MEE" — This is another specialty silicone elastomer which was part of the development of materials for low temperature service. It has excellent consistency over a very broad temperature range—even better than the MEA material described above. The compromise with this elastomer is its low damping level. The typical loss factor for MEE is approximately 0.11 which results in resonant transmissibility in the range of 9.0. The low damping does give this material the desirable feature of providing excellent high frequency isolation characteristics along with its outstanding temperature stability.

With the above background, some of the properties of these elastomers, as they apply to the application of Lord isolators, will be presented. As with metals, elastomers have measureable modulus properties. The stiffness and damping characteristics of isolators are directly proportional to these moduli and vary as the moduli vary.

Strain, Temperature and Frequency Effects — The engineering properties of elastomers vary with strain (the amount of deformation due to dynamic disturbance), temperature and the frequency of the dynamic disturbance. Of these three effects, frequency typically is the least and, for most isolator applications, can normally be neglected. Strain and temperature effects must be considered.

Strain Sensitivity — The general trend of dynamic modulus with strain is that modulus decreases with increasing strain. This same trend is true of the damping modulus. The ratio of the damping modulus to dynamic elastic modulus is approximately equal to the loss factor for the elastomer. The inverse of this ratio may be equated to the expected resonant transmissibility for the elastomer. This may be expressed as:

$$\frac{G''}{G'} \cong \eta$$

$$\frac{G'}{G''} \cong T_R$$

Where: G' **is dynamic modulus (psi)**
G'' **is damping (loss) modulus (psi)**
η **is loss factor**
T_R **is resonant transmissibility**

more exactly:

$$T_R = \sqrt{\frac{1+\eta^2}{\eta^2}}$$

In general, resonant transmissibility varies only slightly with strain while the dynamic stiffness of an isolator may, depending on the elastomer, vary quite markedly with strain.

Figure 5 presents curves which depict the variation of the dynamic modulus of various elastomers which may be used in vibration isolators as related to the dynamic strain across the elastomer. These curves may be used to approximate the change in dynamic stiffness of an isolator due to the dynamic strain across the elastomer. This is based on the fact that the dynamic stiffness of an isolator is directly proportional to the dynamic modulus of the elastomer used in it. This relationship may be written as:

$$K' = \frac{AG'}{t}$$

Where: K' **is dynamic shear stiffness (lb/in)**
G' **is dynamic shear modulus of the elastomer (psi)**
t **is elastomer thickness (in)**
A **is load area of the elastomer (in^2)**

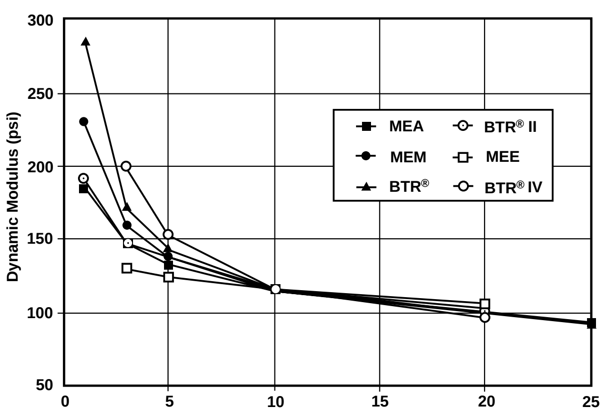


FIGURE 5
TYPICAL DYNAMIC ELASTIC MODULUS VALUES FOR LORD VIBRATION ISOLATOR ELASTOMERS

This variation may be used to calculate the change in a dynamic system's natural frequency from the equation:

$$f_n = 3.13\sqrt{\frac{K'_T}{W}}$$

Where: f_n **is system natural frequency(Hz)**
K'_T **is total system dynamic spring rate (lb/in)**
W **is total weight supported by the isolators**

As there is a change in dynamic modulus, there is a variation in damping due to the effects of strain in elastomeric materials. One indication of the amount of damping in a system is the resonant transmissibility of that system. Figure 6 shows the variation in resonant transmissibility due to changes in vibration input for the elastomers typically used in Lord military electronics isolators.

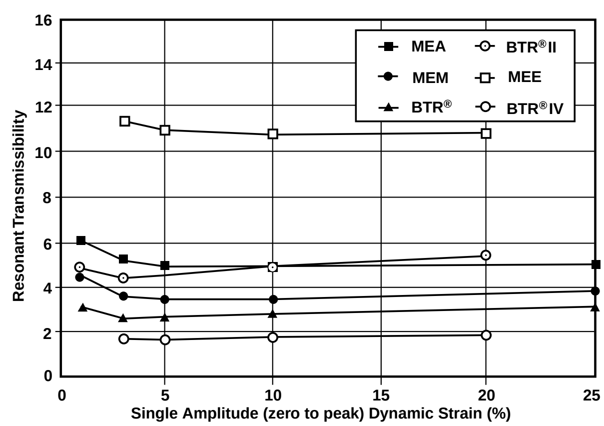


FIGURE 6
TYPICAL RESONANT TRANSMISSIBILITY VALUES FOR LORD VIBRATION ISOLATOR ELASTOMERS

The data presented in Figures 5 and 6 lead to some conclusions about the application of vibration isolators. The following must be remembered when analyzing or testing an isolated system:

- It is important to specify the dynamic conditions under which the system will be tested.
- The performance of the isolated system will change if the dynamic conditions (such as vibration input) change.
- The change in system performance due to changing dynamic environment may be estimated with some confidence.

Temperature Sensitivity — Temperature, like strain, will affect the performance of elastomers and the systems in which elastomeric isolators are used. Figures 7 and 8 show the variations of dynamic modulus and resonant transmissibility with temperature and may be used to estimate system performance changes as may Figures 5 and 6 in the case of strain variation.

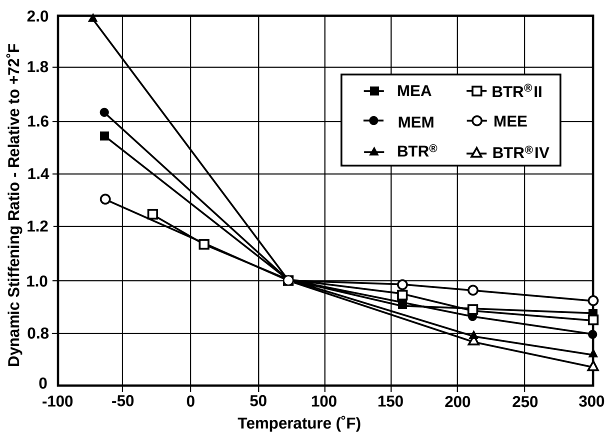


FIGURE 7
TYPICAL TEMPERATURE CORRECTIONS FOR
LORD VIBRATION ISOLATOR ELASTOMERS

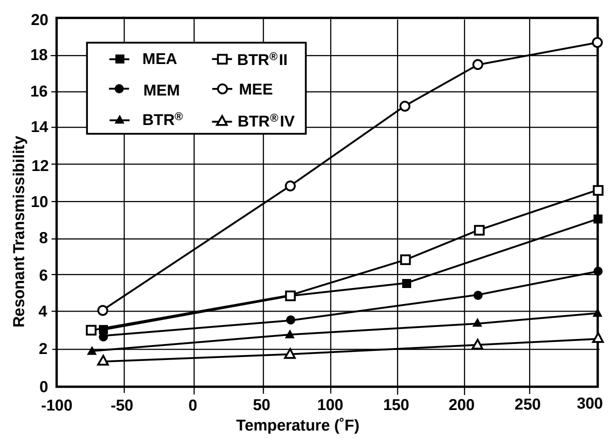


FIGURE 8
TRANSMISSIBILITY VS. TEMPERATURE FOR
LORD VIBRATION ISOLATOR ELASTOMERS

Modifications to Theory Based on the Real World

It should be apparent from the preceding discussion that the basic assumption of linearity in dynamic systems must be modified when dealing with elastomeric vibration isolators. These modifications do affect the results of the analysis of an isolated system and should be taken into account when writing specifications for vibration isolators. It should also be noted that similar effects of variation with vibration level have been detected with "metal mesh" isolators. Thus, care must be exercised in applying them. The amount of variability of these isolators is somewhat different than with elastomeric isolators and depends on too many factors to allow simple statements to be made.

The following discussion will be based on the properties of elastomeric isolators.

Static Stiffness versus Shock Stiffness versus Vibration Stiffnesses — Because of the strain and frequency sensitivity of elastomers, elastomeric vibration and shock isolators perform quite differently under static, shock or vibration conditions.

The equation:

$$d_{static} = \frac{9.8}{f_n^2}$$

Where d_{static} is the "static deflection" of the system (in) and f_n is the system natural frequency (Hz)

DOES NOT HOLD for elastomeric vibration/shock isolators. The static stiffness is typically less than the dynamic stiffness for these materials. To say this another way, the static deflection will be higher than expected if it were calculated, using the above formula, based on a vibration or shock test of the system.

Similarly, neither the static nor the vibration stiffness of such devices is applicable to the condition of shock disturbances of the system. It has been found empirically that:

$$K'_{shock} \cong 1.4K_{static}$$

The difference in stiffness between vibration and static conditions depends on the strain imposed by the vibration on the elastomer. Figure 5 shows where the static modulus will lie in relation to the dynamic modulus for some typical elastomers at various strain levels.

What this means to the packaging engineer or dynamicist is that one, single stiffness value cannot be applied to all conditions and that the dynamic to static stiffness relationship is dependent on the particular isolator being considered. What this means to the isolator designer is that each condition of use must be separately analyzed with the correct isolator stiffness for each condition.

Shock Consideration — As stated in the previous discussion, shock analyses for systems using elastomeric isolators should be based on the guideline that the isolator stiffness will be approximately 1.4 times the static stiffness. In addition to this, it must be remembered that there must be enough free deflection in the system to allow the shock energy to be stored in the isolators. If the system should bottom, the "g" level transmitted to the mounted equipment will be much higher than would be calculated. In short, the system must be allowed to oscillate freely once it has been exposed to a shock disturbance to allow theory to be applied appropriately. Figure 9 shows this situation schematically.

In considering the above, several items should be noted:

- Damping in the system will dissipate some of the input energy and the peak transmitted shock will be slightly less than predicted based on a linear, undamped system.
- "τ" is the shock input pulse duration (seconds)
- "t_n" is one-half of the natural period of the system (seconds)
- There must be enough free deflection allowed in the system to store the energy without bottoming (snubbing). If this is not considered, the transmitted shock may be significantly higher than calculated and damage may occur in the mounted equipment.

Vibration Considerations — The performance of typical elastomeric isolators changes with changes in dynamic input—the level of vibration to which the system is being subjected. This is definitely not what most textbooks on vibration would imply. The strain sensitivity of the elastomers causes the dynamic characteristics to change.

Figure 10 is representative of a model of a vibratory system proposed by Professor Snowdon of Penn State University in his book, "Vibration and Shock in Damped Mechanical Systems." This model recognized the changing properties of elastomers and the effects of these changes on the typical vibration response of an isolated system. These effects are depicted in the comparison of a theoretically calculated transmissibility response curve to one resulting from a test of an actual system using elastomeric isolators.

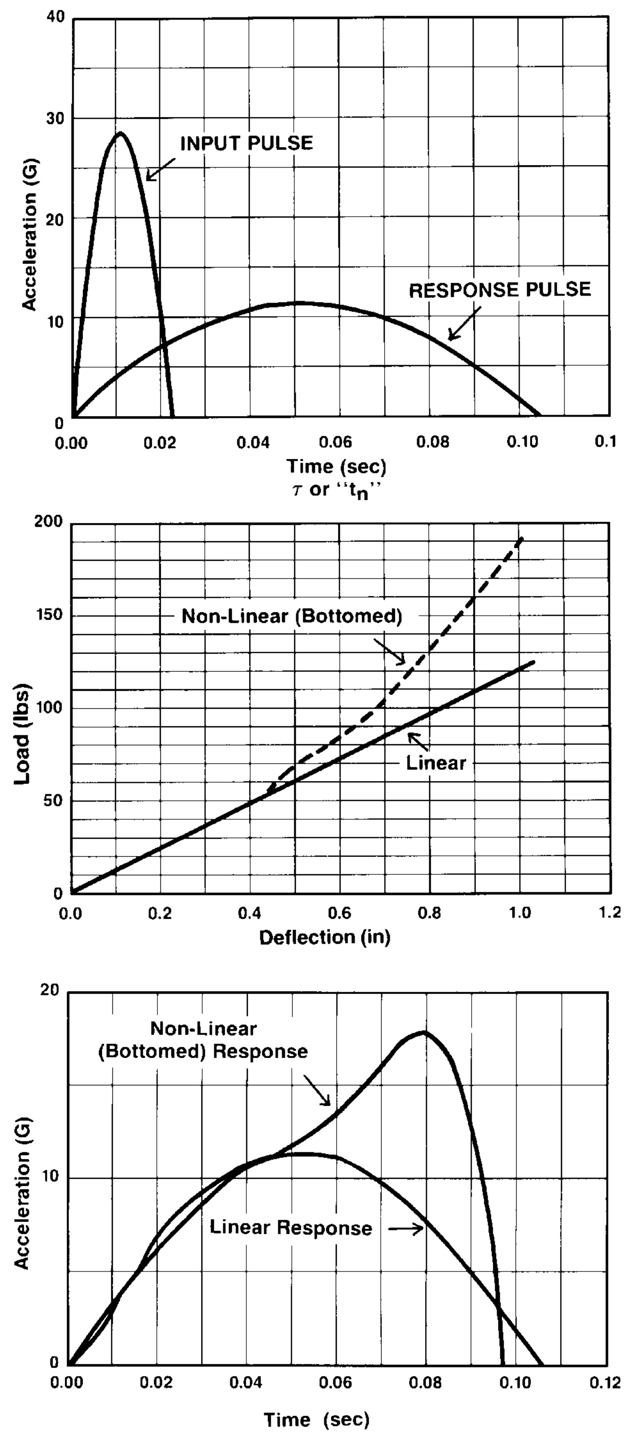


FIGURE 9

The Real World

The majority of vibration and shock isolators are those utilizing elastomeric elements as the source of compliance and damping to control system responses.

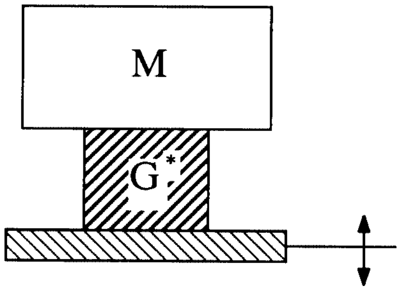


FIGURE 10

G^* is "Complex Modulus"

$$G^* = G' + jG''$$
$$\text{or } G^* = G'(1 + j\eta)$$

Where "η" is loss factor

$$\eta \cong \frac{G''}{G'} \cong 2\zeta$$

G'' is Damping Modulus (psi)
G' is Dynamic Modulus (psi)
and ζ is damping factor (dimensionless)

Using this model, we may express the absolute transmissibility of the system as:

$$T_{ABS} = \frac{\sqrt{1+\eta^2}}{\sqrt{\left[1 - r^2 \frac{G'}{G_{n}'}\right]^2 + \eta^2}}$$

Where G_n' is Dynamic Modulus (psi) at the particular vibration condition being analyzed.

The resulting transmissibility curve from such a treatment, compared to the classical, theoretical transmissibility curve, is shown in Figure 11.

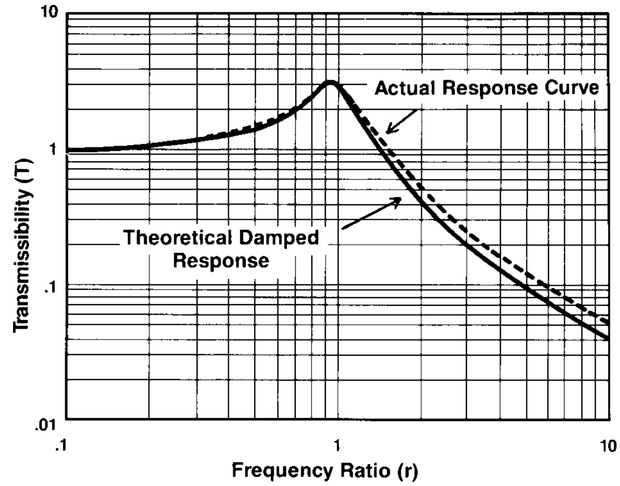


FIGURE 11
EFFECT OF MATERIAL SENSITIVITY ON TRANSMISSIBILITY RESPONSE

Two important conclusions may be reached on the basis of this comparison:

1. The "crossover" point of the transmissibility curve **(T_{ABS} = 1.0)** occurs at a frequency higher than $\sqrt{2}$ times the natural frequency which is what would be expected based on classical vibration theory. This crossover frequency will vary depending on the type of vibration input and the temperature at which the test is being conducted.
2. The degree of isolation realized at high frequencies **(T_{ABS} < 1.0)** will be less than calculated for an equivalent level of damping in a classical analysis. This slower "roll-off" rate $\left(\frac{\text{db}}{\text{octave}}\right)$ will depend, also, on the type of elastomer, level and type of input and temperature.

In general, a constant amplitude sinusoidal vibration input will have less effect on the transmissibility curve than a constant 'g' (acceleration) vibration input. The reason is that, with increasing frequency, the strain across the elastomer is decreasing more rapidly with the constant 'g' input than with a constant amplitude input. Remembering the fact that decreasing strain causes increasing stiffness in elastomeric isolators, this means that the crossover frequency will be higher and the roll-off rate will be lower for a constant 'g' input than for a constant amplitude input. Figure 12 is representative of these two types of vibration inputs as they might appear in a test specification.

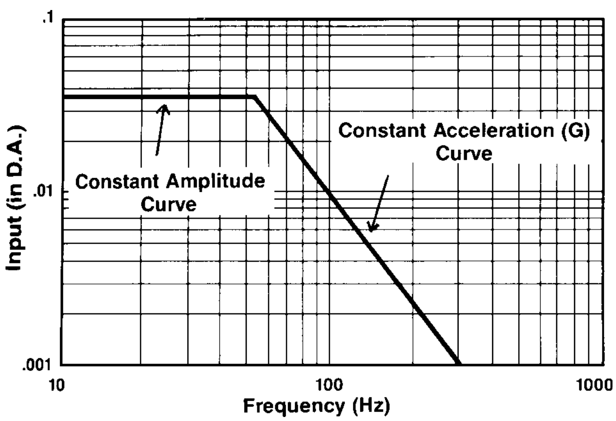


FIGURE 12
COMPARISON-CONSTANT AMPLITUDE TO CONSTANT "G" VIBRATION INPUT

No general statement of where the effects of random vibration will lead in relationship to a sinusoidal constant 'g' or constant amplitude vibration input can be made. However, the effects will be similar to a sinusoidal vibration since random vibrations typically produce lower strains across isolators as frequency increases. There may be some exceptions to this statement. The section titled, "Determining Necessary Characteristics of Vibration/Shock Isolator" provides guidance as to how to apply the properties of elastomers to the various conditions which may be specified for a typical installation requiring isolators.

Data Required to Select or Design a Vibration/ Shock Isolator — As with any engineering activity, the selection or design of an isolator is only as good as the information on which that selection or design is based. Figure 13 is an example of one available Lord checklist for isolator applications — Document number SI-6106.

If the information on this checklist is provided, the selection of an appropriate isolator can be aided greatly, both in timeliness and suitability.

Section I provides the information about the equipment to be mounted (its size, weight and inertias) and the available space for the isolation system to do its job. This latter item includes isolator size and available sway space for equipment movement.

Section II tells the designer what the dynamic disturbances are and how much of those disturbances the equipment can withstand. The difference is the function of the isolation system.

It is important to note here that the random vibration must be provided as a power spectral density versus frequency tabulation or graph, not as an overall "g_{rms}" level, in order to allow analysis of this condition. Also, note that the U.S. Navy "high impact" shock test is required by specification MIL-S-901 for shipboard equipment.

Section III contains space for descriptions of any special environmental exposures which the isolators must withstand. Also, for critical applications, such as gyros, optics and radar isolators, the requirements for control of angular motion of the isolated equipment are requested. In such cases, particular effort should be made to keep the elastic center of the isolation system and the center of gravity of the equipment at the same point. The vibration isolators may have their dynamic properties closely matched in order to avoid the introduction of angular errors due to the isolation system itself.

All of the information listed on the checklist shown in Figure 13 is important to the selection of a proper vibration isolator for a given application. As much of the information as possible should be supplied as early as possible in the design or development stage of your equipment. Of course, any drawings or sketches of the equipment and the installation should also be made available to the vibration/shock analyst who is selecting or designing isolators.

Determining Necessary Characteristics of a Vibration/Shock Isolator

The fragility of the equipment to be isolated is typically the determining factor in the selection or design of an isolator. The critical fragility level may occur under vibration conditions or shock conditions. Given one of these starting points, the designer can then determine the dynamic properties required of isolators for the application. Then, knowing the isolator required, the designer may estimate the remaining dynamic and static performance properties of the isolator and the mounted system.

The following sections will present a method for analyzing the requirements for an isolation problem and for selecting an appropriate isolator.

Sinusoidal Vibration Fragility as the Starting Point — A system specification, equipment operation requirements or a known equipment fragility spectrum may dictate what the system natural frequency must, or may, be. Figure 14 shows a fictitious fragility curve superimposed on a typical vibration input curve. Isolation system requirements may be derived from this information.

SAMPLE

Engineering Data For Vibration and Shock Isolator Questionnaire

For actual questionnaire, see page 99. Please fill in as much detail as possible before contacting Lord. You may mail, fax or e-mail this completed form.

For Technical Assistance, Contact: Application Support, Aerospace Engineering, Lord Corporation, Mechanical Products Division, 2000 W. Grandview Blvd., Erie, PA 16514; Phone: 814/868-0924, Ext. 6611 or 6497; FAX: 814/864-5468; E-mail: apsupport@lord.com

I. Physical Data

A. Equipment weight ____________________

B. C.G. location relative to mounting points ____________________

C. Sway space ____________________

D. Maximum mounting size ____________________

E. Equipment and support structure resonance frequencies ____________________

F. Moment of inertia through C.G. for major axes (necessary for natural frequency and coupling calculations)
I xx ____________ I yy ____________ I zz ____________

G. Fail-safe installation required? Yes ☐ No ☐

II. Dynamics Data

A. Vibration requirement:

1. Sinusoidal inputs (specify sweep rate, duration and magnitude or applicable input specification curve) ____________________
2. Random inputs (specify duration and magnitude (g^2/Hz) applicable input specification curve)
____

B. Resonant dwell (input & duration) ____________________

C. Shock requirement:

1. Pulse shape ____________ pulse period ____________ amplitude ____________
number of shocks per axis ____________ maximum output ____________
2. Navy hi impact required? (if yes, to what level?)____________________

D. Sustained acceleration: magnitude ____________ direction ____________
Superimposed with vibration? Yes ☐ No ☐

E. Vibration fragility envelope (maximum G vs. frequency preferred) or desired natural frequency and maximum transmissibility ____________________

F. Maximum dynamic coupling angle ____________________
matched mount required? Yes ☐ No ☐

G . Desired returnability ____________________
Describe test procedure ____________________

III. Environmental Data

A. Temperature: Operating ____________ Non-operating ____________

B. Salt spray per MIL ____________ Humidity per MIL ____________
Sand and dust per MIL ____________ Fungus resistance per MIL ____________
Oil and/or gas ____________ Fuels ____________

C. Special finishes on components ____________________

FIGURE 13

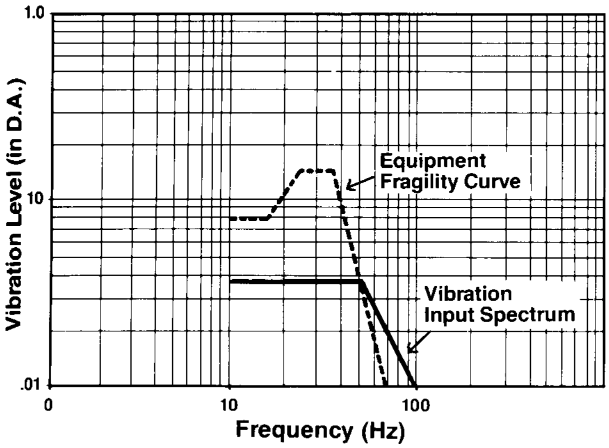


FIGURE 14
EQUIPMENT FRAGILITY VS. VIBRATION INPUT

First, the allowable transmissibility at any frequency may be calculated as the ratio of the allowable output to the specified input.

$$T_{ABS} = \frac{X_o}{X_i} \text{ or } \frac{g_o}{g_i}$$

The frequency at which this ratio is a maximum is one frequency at which the system natural frequency may be placed (assuming that it is greater than approximately 2.5, at some frequency). Another method of placing the system natural frequency is to select that frequency which will allow the isolation of the input over the required frequency range. A good rule of thumb is to select a frequency which is at least a factor of 2.0 below that frequency where the allowable response (output) crosses over — goes below — the specified input curve.

Having determined an acceptable system natural frequency, the system stiffness (spring rate) may be calculated from the following relationship:

$$K'_v = \frac{(f_n)^2(W)}{9.8}$$

Where: K'_V is the total system dynamic stiffness (lb/in) at the specified vibration input
f_n is the selected system natural frequency (Hz)
W is the isolated equipment weight (lbs)

An individual isolator spring rate may then be determined by dividing this system spring rate by the allowable, or desired number of isolators to be used. The appropriate isolator may then be selected based on the following factors:

- required dynamic spring rate
- specified vibration input at the desired natural frequency of the system
- static load supported per isolator
- allowable system transmissibility
- environmental conditions (temperature, fluid exposure, etc.)

Once a particular isolator has been selected, the properties of the elastomer in the isolator may be used to estimate the performance of the isolator at other conditions of use, such as other vibration levels, shock inputs, steady state acceleration loading and temperature extremes. The necessary elastomer property data are found in Figures 5, 6, 7 and 8.

If the vibration input in the region of the required natural frequency is specified as a constant acceleration—constant ‘g’—it may be converted to a motion input through the equation:

$$X_i = \frac{g_i}{(0.051)(f_n)^2}$$

Where: X_i is vibratory motion (inches, double amplitude)
g_i is specified vibratory acceleration input (g)
f_n is the desired system natural frequency (Hz)

Of course, this equation may be used to convert constant acceleration levels to motions at any frequency. It is necessary to know this vibratory motion input in order to select or design an isolator. Note, that most catalog vibration isolators are rated for some maximum vibration input level expressed in inches double amplitude. Also, the listed dynamic stiffnesses for many standard isolators are given for specific vibration inputs. This information provides a starting point on Figure 5 to allow calculation of the system performance at vibration levels other than that listed for the isolator.

Random Vibration Performance as the Starting Point — Random vibration is replacing sinusoidal vibration in specifications for much of today’s equipment. A good example is MIL-STD-810. Many of the vibration levels in the most recent version of this specification are given in the now familiar format of “power spectral density” plots. Such specifications are the latest attempt to simulate the actual conditions facing sensitive equipment in various installations.

A combination of theory and experience is used in the analysis of random vibration. As noted previously, the random input must be specified in the units of “g^2/Hz” in order to be analyzed and to allow proper isolator selection. The system natural frequency may be determined by a fragility versus input plot of random vibration just as was done and demonstrated in Figure 14 for sinusoidal vibration. Once the required natural frequency is known, the necessary isolator spring rate may again be calculated from the equation:

$$K'_v = \frac{(f_n)^2(W)}{9.8}$$

The next steps in determining which isolator may be used are to calculate the allowable transmissibility and the motion at which the isolated system responds at the same natural frequency as when it is subjected to the specified random vibration. The allowable transmissibility, if not already specified, may be calculated from the input vibration and the allowable vibration by using the equation:

$$T_R = \sqrt{\frac{S_o}{S_i}}$$

Where, T_R is the resonant transmissibility (dimensionless)
S_O is output random vibration (g^2/Hz)
S_i is input random vibration (g^2/Hz)

A sinusoidal vibration input, acceleration or motion, at which the system will respond at approximately the same natural frequency with the specified random vibration may be calculated in the following manner.

Step 1. The analysis of random vibration is made on the basis of probability theory. The one sigma **(1σ)** RMS acceleration response may be calculated from the equation:

$$g_{oRMS} = \sqrt{(\pi/2)(S_i)(f_n)T_R}$$

Where, g_oRMS is the 1σ RMS acceleration response (g)
S_i is input random vibration (g2/Hz)
T_R is allowable resonant transmissibility
f_n is desired natural frequency (Hz)

Step 2. It has been found empirically that elastomeric isolators typically respond at a **3**σ vibration level. Thus, the acceleration vibration level at which the system will respond at approximately the same natural frequency as with the specified random level may be found to be:

$$g_{3\sigma} = 3\sqrt{(\pi/2)(S_i)(f_n)T_R}$$

Step 3. The above is response acceleration. To find the input for this condition of response, we simply divide by the resonant transmissibility.

$$g_i = \frac{g_{3\sigma}}{T_R}$$

Step 4. Finally, we apply the equation from a previous section to calculate the motion input vibration equivailent to this acceleration at the system natural frequency:

$$X_i = \frac{g_i}{(0.051)(f_n)^2}$$

Note that X_i is in units of inches double amplitude.

Step 5. The analysis can now follow the scheme of previous calculations to find the appropriate isolator and then analyze the shock, static and temperature performance of the isolator.

Shock Fragility as the Starting Point —If the fragility of the equipment in a shock environment is the critical requirement of the application, the natural frequency of the system will depend on the required isolation of the shock input.

Step 1. Calculate the necessary shock transmissibility

$$T_S = \frac{g_o}{g_i}$$

Where T_s is shock transmissibility (dimensionless)
g_o is equipment fragility (g)
g_i is input shock level (g)

Step 2. Calculate the required shock natural frequency. This depends on the shape of the shock pulse.

The following approximate equations may be used only for values of $T_s < 1.0$:

Pulse Shape	Transmissibility Equation
Half Sine	$Ts \cong 4(f_n)(t_o)$
Square Wave	$Ts \cong 6(f_n)(t_o)$
Triangular	$Ts \cong 3.1(f_n)(t_o)$
Ramp or Blast	$Ts \cong 3.2(f_n)(t_o)$

Where T_s is shock transmissibility
f_n is shock natural frequency
t_o is shock pulse length (seconds)

Remember, that the system natural frequency under a shock condition will typically be different from that under a vibration condition for systems using elastomeric vibration isolators.

Step 3. Calculate the required deflection to allow this level of shock protection by the equation:

$$d_s = \frac{g_o}{(0.102)(f_n^2)}$$

Where d_s is shock deflection (inches Single Amplitude)
g_o is shock response or equipment fragility (g)
f_n is shock natural frequency (Hz)

Step 4. Calculate the required dynamic spring rate necessary under the specified shock condition from the equation:

$$K'_s = \frac{(f_n)^2 W}{9.8}$$

Where K'_s is dynamic stiffness (lb/in)
f_n is shock natural frequency (Hz)
W is supported weight (lbs)

Step 5. Select the proper isolator from those available in the product section, that is, one which has the required dynamic stiffness **(K'_v)**, will support the specified load and will allow the calculated deflection **(d_s)** without bottoming during the shock event.

Step 6. Determine the dynamic stiffness **(K'_v)** of the chosen isolator, at the vibration levels specified for the application, by applying Figure 5 with the knowledge that dynamic spring rate is directly proportional to dynamic modulus **(G')** and by working from a known dynamic stiffness of the isolator at a known dynamic motion input.

Step 7. Calculate system natural frequencies under specified vibration inputs from the equation:

$$f_n = 3.13\sqrt{\frac{K'_v}{W}}$$

Where f_n is vibration natural frequency (Hz)
K'_v is isolator dynamic stiffness at the specified vibration level (lbs/in)
W is the supported weight (lbs)

Note that the stiffness and supported weight must be considered on the same terms, i.e., if the stiffness is for a single mount, then the supported weight must be that supported on one mount. Once the system natural frequency is calculated, the system should be analyzed to determine what effect this resonance will have on the operation and/or protection of the equipment.

Step 8. Estimate the static stiffness of the isolators from the relationship:

$$K \cong \frac{K'_s}{1.4}$$

Where K is static stiffness (lbs/in)
K'_s is shock dynamic stiffness (lbs/in)

Then, check the deflection of the system under the 1g load and under any steady-state (maneuver) loads from the equation:

$$d_s = \frac{gW}{K}$$

Where d_s is static deflection (inches)
g is the number of g's loading being imposed
W is the supported load (lbs)
K is static spring rate (lbs/in)

Be sure that the chosen isolator has enough deflection capability to accommodate the calculated motions without bottoming. If the vibration isolation function and steady state accelerations must be imposed on the system simultaneously, the total deflection capability of the isolator must be adequate to allow the deflections from these two sources combined. Thus,

$$d_{total} = d_v + d_s$$

$$\text{where } d_v = \frac{x_i}{2} T_R$$

and where x_i is input vibration motion at resonance (inches double amplitude)
d_v is deflection due to vibration (inches single amplitude)
T_R is resonant transmissibility
d_s is static deflection per the above equation (inches)

Types of Isolators and Their Properties — There are a number of different types of isolators, based on configuration, which may be applied in supporting and protecting various kinds of equipment. Depending on the severity of the application and on the level of protection required for the equipment, one or another of these mounting types may be applied.

Figures 15, 16 and 17 show some of the most common "generic" configurations of vibration isolators and the characteristic load versus deflection curves for the simple shear mounting and the "buckling column" types of isolators. In general, the fully bonded or holder types of isolators are used for more critical equipment installations because these have superior performance characteristics as compared to the center bonded or unbonded configurations. The buckling column type of isolator is useful in applications where high levels of shock must be reduced in order to protect the mounted equipment. Many aerospace equipment isolators are of the conical type because they are isoelastic.

In order of preference for repeatability of performance the rank of the various isolator types is:

1. Fully Bonded
2. Holder Type
3. Center Bonded
4. Unbonded

In reviewing the standard lines of Lord isolators, the STANDARD AVIONICS (AM), PEDESTAL (PS), PLATEFORM (100,106,150,156), HIGH DEFLECTION (HDM) and MINIATURE (MAA) mounts are in the fully bonded category. The BTR (HT) mounts are the only series in the holder type category. The MINIATURE (MCB) series of isolators is the offering in the center bonded type of mount. The MINIATURE GROMMETS (MGN and MGS) are in the unbonded mount category. In total, these standard offerings from Lord cover a wide range of stiffnesses and load ratings to satisfy the requirements of many vibration and shock isolation applications.

In some instances, there may be a need to match the dynamic stiffness and damping characteristics of the isolators which are to be used on any particular piece of equipment. Some typical applications of matched sets of isolators are gyros, radars and optics equipment. For these applications, the fully bonded type of isolator construction is highly recommended. The dynamic performance of these mounts is much more consistent than other types. Dynamically matched isolators are supplied in sets but are not standard since matching requirements are rarely the same for any two applications.

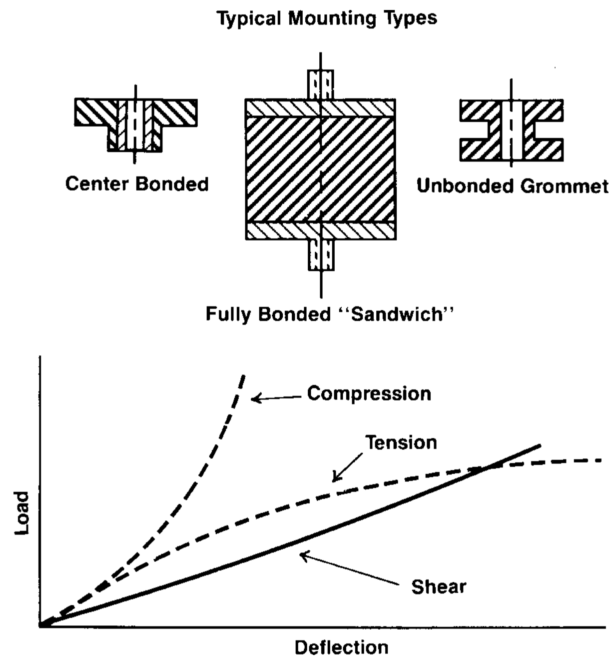


FIGURE 15
LOAD-DEFLECTION CURVES FOR "SANDWICH" MOUNTS

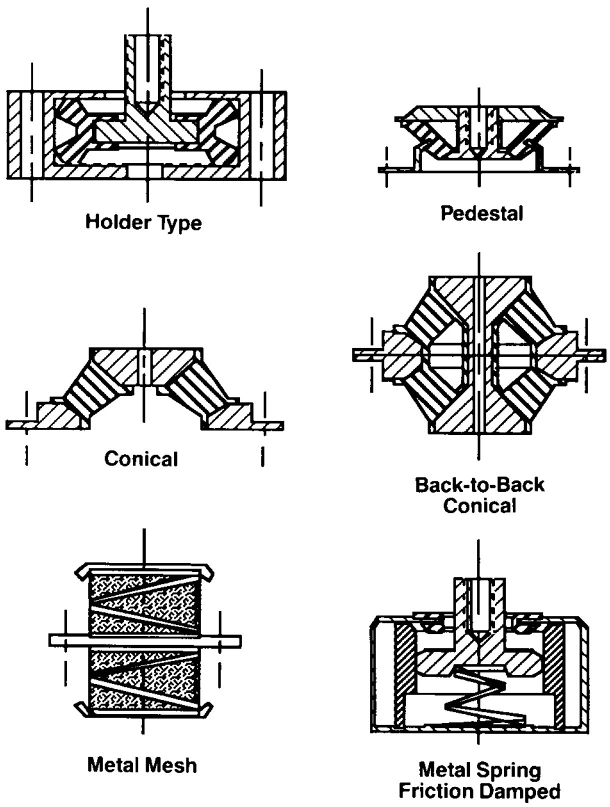


Figure 16

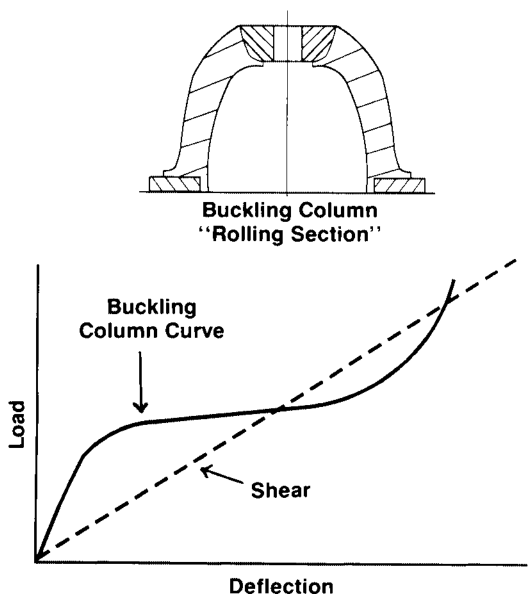


Figure 17

Sample Application Analysis — Figure 18 is a completed checklist of information for a fictitious piece of Avionics gear installed in an aircraft environment. The following section will demonstrate how the foregoing theory and data may be applied to the selection of a standard Lord mount.

CONSIDER SINUSOIDAL VIBRATION REQUIREMENTS

From the checklist, it is noted that the desired system natural frequency is 32 Hz with a maximum allowable transmissibility of 4.0, or less.

Step 1. Determine the required dynamic spring rate:

$$K'_v = \frac{(f_n)^2(W)}{9.8}$$

$$f_n = 32\ \text{Hz}$$

$$W = 12\ \text{lbs}$$

$$K'_v = \frac{(32)^2(12)}{9.8} = 1254\ \text{lbs/in}$$

Note that this figure is the total system spring rate since the weight used in the calculation was the total weight of the supported equipment. The checklist indicates that four (4) isolators will be used to support this unit. Thus, the required isolator is to have a dynamic stiffness of:

$$K'_v = \frac{1254}{4} = 314\ \text{lbs/in/isolator}$$

at the vibration input of 0.036 inch double amplitude as specified in section II.A.1 of the checklist.

Step 2. Make a tentative isolator selection.

Thus far, it is known that:

1. The isolator must have a dynamic spring rate of 314 lbs/in.
2. The supported static load per isolator is 3 pounds.
3. The material, or construction, of the isolator must provide enough damping to control resonant transmissibility to 4.0 or less.
4. There is no special environmental resistance required.

Choosing a relatively small isolator available from those which meet the above requirements, the AM003-7, in BTR® elastomer, is selected from the product data section. The analysis now proceeds to consideration of other specified conditions.

SAMPLE

Engineering Data For Vibration and Shock Isolators Questionnaire

For actual questionnaire, see page 99. Please fill in as much detail as possible before contacting Lord. You may mail, fax or e-mail this completed form.

For Technical Assistance, Contact: Application Support, Aerospace Engineering, Lord Corporation, Mechanical Products Division, 2000 W. Grandview Blvd., Erie, PA 16514; Phone: 814/868-0924, Ext. 6611 or 6497; FAX: 814/864-5468; E-mail: apsupport@lord.com

I. Physical Data

A. Equipment weight 12 lbs.

B. C.G. location relative to mounting points Geometric Center
Four Mounts Desired

C. Sway space ± 0.32"

D. Maximum mounting size 1" High x 2" Long x 2" Wide

E. Equipment and support structure resonance frequencies 400 Hz

F. Moment of inertia through C.G. for major axes (necessary for natural frequency and coupling calculations)
(unknown) I xx ______ I yy ______ I zz ______

G. Fail-safe installation required? Yes ☐ No ☒

II. Dynamics Data

A. Vibration requirement:

1. Sinusoidal inputs (specify sweep rate, duration and magnitude or applicable input specification curve)
.036" D.A. 5 to 52 Hz; 5G, 52 to 500 Hz
2. Random inputs (specify duration and magnitude (g^2/Hz) applicable input specification curve)
.04 G^2/Hz 10 to 300 Hz;

B. Resonant dwell (input & duration) .036" D.A. 1/2 hr. per Axis

C. Shock requirement:

1. Pulse shape Half Sine pulse period 11ms amplitude 15G
number of shocks per axis 3/Axis maximum output N/A
2. Navy hi impact required? N/A (if yes, to what level?)

D. Sustained acceleration: magnitude 3G direction all directions
Superimposed with vibration? Yes ☒ No ☐

E. Vibration fragility envelope (maximum G vs. frequency preferred) or desired natural frequency and maximum transmissibility 32 Hz with T less than 4

F. Maximum dynamic coupling angle N.A.
matched mount required? Yes ☐ No ☐

G. Desired returnability N.A.
Describe test procedure N.A.

III. Environmental Data

A. Temperature: Operating +30° to +120°F Non-operating -40° to +160°F

B. Salt spray per MIL 810C Humidity per MIL 810C
Sand and dust per MIL 810C Fungus resistance per MIL 810C
Oil and/or gas N.A. Fuels N.A.

C. Special finishes on components N.A.

FIGURE 18

Consider Random Vibration Requirements

Step 1. Calculate a sinusoidal motion input at the desired natural frequency with the specified random vibration input and compare it to the specified sine vibration. Both the maximum motion and the input motion which would cause the isolator to respond at approximately the same natural frequency as the random vibration should be calculated. The maximum is calculated to check that the selected isolator will have enough deflection capability and the resonant motion is calculated to verify the stiffness of the required isolator at the actual input at which it will respond to the random vibration.

Per the previously presented material, the isolator should respond at a 3σ equivalent acceleration — calculated on the basis of the specified random vibration at the desired natural frequency. This level will determine, in part, the isolator choice. The calculation is made as follows:

$$g_{o3\sigma} = 3\sqrt{(\pi/2)(S_i)(f_n)(T_R)}$$

In which: S_i **= 0.04 g2/Hz**
T_R **= 2.9 (per Figure 6 for BTR® at typical operating strain)**
f_n **= 32 Hz**

$$g_{o3\sigma} = 3\sqrt{(\pi/2)(0.04)(32)(2.9)}$$

$$g_{o3\sigma} = 7.24g$$

This is the acceleration response at the desired natural frequency of 32 Hz. The motion across the isolator due to this response may be calculated as:

$$x_{o3\sigma} = g_{o3\sigma}/(0.051)(f_n^2)$$

$$x_{o3\sigma} = 7.24/(0.051)(32^2)$$

$$x_{o3\sigma} = 0.139 \text{ inch double amplitude}$$

The ultimately selected isolator must have enough deflection capability to allow this motion without bottoming (snubbing). The input acceleration is calculated as:

$$g_{i3\sigma} = g_{o3\sigma}/T_R$$

$$g_{i3\sigma} = 7.24/2.9$$

$$g_{i3\sigma} = 2.5g$$

and the input motion as:

$$x_{i3\sigma} = g_{i3\sigma}/(0.051)(f_n^2)$$

$$x_{i3\sigma} = 2.5/(0.051)(32^2)$$

$$x_{i3\sigma} = 0.048 \text{ inch double amplitude}$$

This vibration level is higher than the capability of the tentatively selected AM003-7. To remain with a relatively small isolator which will support 3 pounds, withstand the 0.047 inch double amplitude sine vibration and provide an approximate stiffness of 314 lb/in per mounting point, a selection from either the AM002 or AM004 series appears to be best.

Since none of the single isolators provides enough stiffness, a back to back (parallel) installation of a pair of isolators at each mounting point is suggested. Since the AM002 is smaller than the AM004, and is rated for 0.06 inch double amplitude maximum input vibration, the selection of the AM002-8 isolator is made. A pair of the AM002-8 isolators will provide a stiffness of 346 lb/inch (two times 173 per the stiffness chart in the product section). This stiffness would provide a slightly higher natural frequency than desired. However, there is a correction to be made, based on the calculated vibration input.

The stiffnesses in the AM002 product chart are based on an input vibration of 0.036 inch double amplitude. Figure 5 shows that the modulus of the BTR® elastomer is sensitive to the vibration input. The modulus is directly proportional to the stiffness of the vibration isolator. Thus, the information of Figure 5 may be used to estimate the performance of an isolator at an "off spec" condition. A simple graphical method may be used to estimate the performance of an isolator at such a condition.

Knowing the geometry of the isolator, the strain at various conditions may be estimated. The modulus versus strain information of Figure 5 and the knowledge of the relationship of modulus to natural frequency (via the stiffness of the isolator) are used to construct the graph of the isolator characteristic. The equation for calculation of the 3σ random equivalent input at various frequencies has been shown previously. The crossing point of the two lines on the graph shown in Figure 19 is a reasonable estimate for the response natural frequency of the selected isolator under the specified 0.04 g2/Hz random vibration.

The intersection of the plotted lines in Figure 19 is at a frequency of approximately 32 to 33 Hz, and at an input vibration level of approximately 0.047 inch DA. This matches the desired system natural frequency and confirms the selection of the AM002-8 for this application. In all, eight (8) pieces of the AM002-8 will be used to provide the 32 Hz system natural frequency, while supporting a total 12 lb unit, under the specified random vibration of 0.04 g2/Hz. The eight isolators will be installed in pairs at four locations. With this portion of the analysis complete, the next operating condition - shock - is now considered.

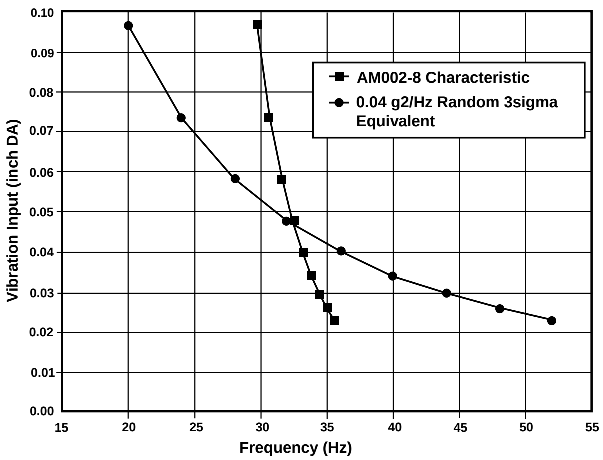


FIGURE 19

Consider Shock Requirements

The specified shock input is a 15g, 11 millisecond, half-sine pulse. From the previously presented theory, an approximation of the shock response may be found through the use of the equation:

$$T_s \cong 4 f_n t_o$$

Note that the natural frequency to be used here is the shock natural frequency which may be estimated from the information given in Figure 5. The dynamic modulus for the elastomer used here is approximately 120 psi at a vibration level of 0.036 inch double amplitude and the static modulus is approximately 80 psi. From this information, the static stiffness of the isolator may be estimated as follows:

$$K = \left(\frac{80}{120}\right)(K')$$

$$K = \left(\frac{80}{120}\right)\left(\frac{f_n{}^2 W}{9.8}\right)$$

$$K = \left(\frac{80}{120}\right)\left(\frac{(32)^2(12)}{9.8}\right) = 836 \text{ lbs/in for the total system}$$

As noted in previous discussion, the shock stiffness is approximately 1.4 times the static stiffness. Thus,

$$K'\text{shock} \cong (1.4)(836) = 1170 \text{ lbs/in total}$$

This makes the shock natural frequency:

$$f_{shock} = 3.13\sqrt{\frac{1170}{12}} = 31 \text{ Hz}$$

Thus, the calculation for the shock transmissibility becomes:

$$T_s \cong (4)(31)(.011) = 1.4$$

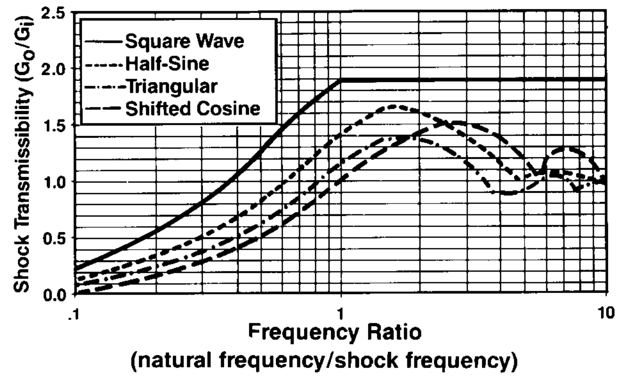


FIGURE 20
SINGLE DEGREE OF FREEDOM SYSTEM
RESPONSE TO VARIOUS SHOCK PULSES

Since this value is above 1.0, and the equation is only valid up to a value of 1.0, the information of Figure 20 must be used. Use of this graph indicates that the shock transmissibility will be approximately 1.22. Thus, the shock response will be:

$$g_o = T_s(g_i)$$

$$G_o = (1.22)(15) = 18.3\,g$$

From this response, the next step is to calculate the expected deflection when the selected isolator is subjected to the specified shock input. The equation of interest is:

$$d_s = \frac{g_o}{(0.102)(f_n)^2}$$

$$ds = \frac{18.3}{(0.102)(31)^2} = 0.19 \text{ inch single amplitude}$$

The tentatively selected isolator, AM002-8, is capable of this much deflection without bottoming. Thus, the analysis proceeds to another operating condition.

Consider "Static" Loading Conditions: The static loading conditions in an isolator analysis are important from the standpoints of stress and deflection to which the isolator will be exposed. Such conditions are caused by the 1g load which the isolator must support as well as by any maneuver and/or steady-state accelerations, which may be imposed. In the present example, the static system stiffness was calculated as being 836 lbs/in. The deflection of the system at any steady-state "g" loading may be calculated by using the equation:

$$d_{static} = \frac{(g)(W)}{K_{static}}$$

In the example, the sustained acceleration was specified as being 3g. Thus, the system deflection will be approximately:

$$d_{static} = \frac{(3)(12)}{836} = 0.043 \text{ inch}$$

The selected isolator, AM002-8, is able to accommodate this deflection, even superimposed on the vibration conditions. Finally, none of the environmental conditions shown on the checklist will be of any concern. Thus, this appears to be an appropriate isolator selection. Of course, typical testing of this equipment, supported by the selected isolators, should be conducted to prove the suitability of this system.

The isolators presented in the product portion of this catalog will prove appropriate for many equipment installations. Should one of these products not be suitable, a custom design may be produced. Lord is particularly well equipped to provide engineering support for such opportunities. For contact information, see page 103. The following brief explanation will provide a rough sizing method for an isolator.

Estimating Isolator Size: There will be occasions when custom designs will be required for vibration and shock isolators. It should be remembered that schedule and economy are in favor of the use of the standard isolators shown in the product section here. These products should be used wherever possible. Where these will not suffice, Lord will assist by providing the design of a special mount. The guidelines presented here are to allow the packaging or equipment engineer to estimate the size of the isolator so that the equipment installation can be made with the thought in mind to allow space for the isolators and for the necessary deflection of the system as supported on them. The final isolator size may be slightly larger or smaller depending on the specifications being imposed.

Figure 21 shows a schematic of a conical isolator, such as may be used for protection of avionic equipment. The two most important parameters in estimating the size of such an isolator are the length of the elastomer wall, $\mathbf{t_R}$, and the available load area. For purposes of simplification, a conical angle of 45° is used here. The ratio of axial to radial stiffness depends on this angle.

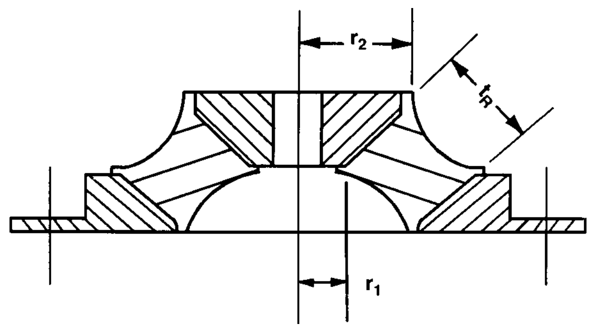


FIGURE 21
ESTIMATING AVIONICS ISOLATOR SIZE

The elastomer wall length may be estimated based on the dynamic motion necessary for the requirements of the application. This length may be estimated through the following equation:

$$t_R = \frac{(x_i)(T_R)}{0.30}$$

Where t_R is the elastomer wall length (inches)
x_i is the resonant vibration input (inches, double amplitude)
T_R is resonant transmissibility

From the required natural frequency, the necessary dynamic spring rate is known from:

$$K' = \frac{(fn)^2(W)}{9.8} \text{ lb/in}$$

Where K' is dynamic stiffness (lb/in)
f_n is desired natural frequency (Hz)
W is supported weight per isolator (lbs)

For a conical type isolator, the dynamic spring rate/geometry relationship is:

$$K' = \frac{(A)(G')}{t_R}$$

Where t_R is the elastomer wall per the above and the area term (A) is estimated as:

$$A \cong 1.4\pi(r_2^2 - r_1^2)$$

This area term should be determined such that the dynamic stress at resonance is kept below approximately 40 psi.

$$\sigma = \frac{P}{A} \leq 40 \text{ psi}$$

and

$$P_{max} \cong (g_i)(T_R)W$$

Where g_i is input 'g' level at resonance
T_R is resonant transmissibility
W is supported load per isolator (lbs)

The combination of the elastomer wall length (t_R) and load area (A), estimated from the above, and the required attachment features will provide a good estimate of the size of the isolator required to perform the necessary isolation functions. The proper dynamic modulus is then selected for the isolator from an available range of approximately 90 to 250 psi at a 0.036 inch D.A., vibration input.

Resonant Dwells: The requirement of a "resonant dwell" of isolated equipment is becoming less common in today's world. However, some projects still have such a requirement and it may be noted that many of the products described in the product sections have been exposed to resonant dwell conditions and have performed very well. Isolators designed to the elastomer wall and load area guidelines given above will survive resonant dwell tests without significant damage for systems with natural frequencies below approximately 65 Hz. Systems higher in natural frequency than this require special consideration and Lord engineers should be consulted.

Environmental Resistance: Many of the isolators shown in this catalog are inherently resistant to most of the environments (temperature, sand, dust, fungus, ozone, etc.) required by many specifications. The silicone elastomers are all in this category. One particularly critical area is fluid resistance where special oils, fuels or hydraulic fluids could possibly come into contact with the elastomer. Lord engineering should be contacted for an appropriate elastomer selection.

Testing of Vibration/Shock Isolators: Lord has excellent facilities for the testing of isolators. Electrodynamic shakers having up to eight thousand pound dynamic force capability are used to test many of the isolators designed or selected for customer use. These shakers are capable of sinusoidal and random vibration testing as well as sine-on-random and random-on-random conditions. These machines are also capable of many combinations of shock conditions and are supplemented with free-fall drop test machines. Numerous isolator qualification tests have been performed within the test facilities at Lord.

Further Theory

The preceding discussion presented general theory which is applicable to a broad class of vibration and shock problems. A special class of shock analysis is that which involves drop tests, or specifications, such as with protective shipping containers. This topic is treated in the following pages.

Introduction to Shipping Container Isolator Selection

A special case of shock protection is found in the Shipping Container market. Here, the shock pulses are not defined as previously discussed but are specified in terms of being dropped from some height in a given configuration. Thus, the following discussion is presented.

This information here is presented to assist in the selection of Lord products to protect critical items in their shipping containers. It is intended that, for most applications, a mount from the line of standard Lord Shipping Container Mounts can be selected.

The basics of shock isolation are presented to give the reader an understanding of the effects of assumptions made during analysis of the system. The relationship of shock response to vibration response of the system as well as to the static stiffness characteristics of the mounts is discussed.

The variables which must be considered in the real world application of elastomeric shock mounts are presented. Included is a discussion of stiffness variation with strain and temperature and the effects of this variation on the overall response of the system.

Some basic equations are presented to allow calculation of system response in simple cases. For those instances where more elaborate analysis is required, a checklist of necessary information for a Lord analysis is provided.

Shock Isolation Theory

Although many factors can influence the dynamic response of a shipping container system, we may look at the overall problem as one of energy being imposed on the system. This energy must be stored, or dissipated. The energy stored in the mounts must then be released back to the system in a controlled manner such that the peak forces transmitted are below the critical level (fragility) for the mounted equipment.

With a given weight and geometry for the mounted equipment, the dynamic stiffness of the shock mounts is the adjustable factor at the designer's disposal to provide the desired protection. This stiffness determines the mounted system natural frequency which, in turn, controls the rate at which the energy is returned to the system and the maximum forces which will be imposed on the equipment.

The energy input to the system enters over some time period **(pulse length t_0)** and reaches some maximum force level **F_0)**. Schematically, this would appear as Figure 22 on a force-time curve. The area enclosed under this curve is proportional to the energy.

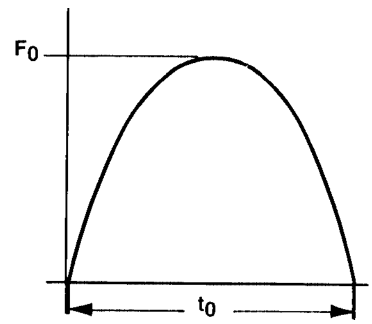


FIGURE 22
FORCE-TIME CURVE—INPUT TO CONTAINER

If the shock mounts are selected correctly to protect the mounted equipment, the response through the mounts will be such that the energy (assuming no dissipation) will be transmitted to the mounted mass over a longer time period than that at which it entered the mounts. With this longer time period, the peak force will be lower than that imposed at the outside of the container. This is shown in Figure 23. Here, the energy is the same as that from Figure 22.

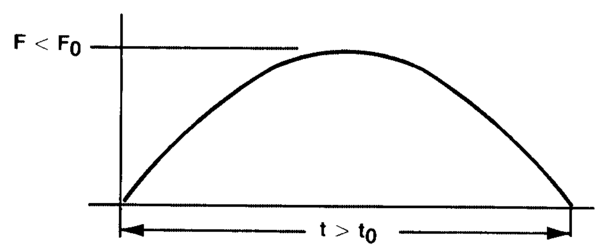


FIGURE 23
FORCE-TIME CURVE—RESPONSE THROUGH SHOCK MOUNTS SHOCK REDUCTION

Conversely, if mounts are incorrectly selected, they could result in amplifying the peak forces seen by the mounted equipment. Figure 24 shows this case. Again, the energy is assumed equivalent to the original energy entering the container.

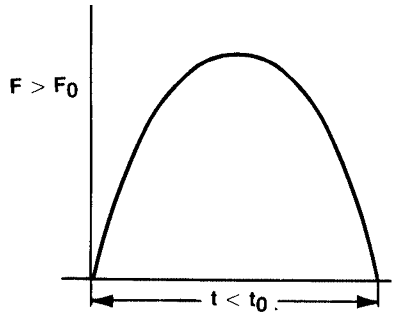


FIGURE 24
FORCE-TIME CURVE—RESPONSE THROUGH
SHOCK MOUNTS SHOCK AMPLIFICATION

It should be noted that the situation of Figure 24 (shock amplification) can occur in a number of ways. Among these are:

• Incorrect mount stiffness

• Non-linear mount stiffness in the necessary deflection range

• Insufficient sway space available within the shipping container.

Thus, it is important to accurately define system parameters, select appropriate shock mounts, and design the shipping container with the mounting system in mind.

Basic Shock Equations:

The basic equations for initial estimates of shock isolation systems are fairly simple. They involve the input to the system and the characteristics of the mounted mass and the shock mounts. In general, the shock to the system is modelled as an instantaneous velocity change for most shipping container applications.

We start the analysis knowing the impact velocity of the container into the barrier or floor. Typically, the velocity for a side or end impact is specified. For drop tests, this velocity must be calculated.

For a straight, vertical drop:

$$V_o = \sqrt{2gH} \qquad (1)$$

Where V_o = impact velocity (in/sec)
g = acceleration due to gravity (386 in/sec^2)
H = drop height (in)

The next necessary item to know is the system natural frequency:

$$f_n = 3.13\sqrt{K'/W} \qquad (2)$$

Where f_n = system natural frequency (Hz)
K' = system dynamic spring rate (lb/in)
W = supported weight (lbs)

Then the response acceleration may be calculated:

$$A_o = \frac{V_o f_n}{61.4} \qquad (3)$$

Where A_o = response acceleration (G)
V_o = impact velocity (in/sec)
f_n = system natural frequency (Hz)

as well as the deflection across the shock mounts:

$$d_o = \frac{9.8A_o}{(f_n)^2} \qquad (4)$$

Where d_o = system deflection (inches)
A_o = response acceleration (G)
f_n = system natural frequency (Hz)

Of course, equation (3) may be solved in reverse if the equipment fragility is known and the system natural frequency is required.

$$f_n = \frac{A_o(61.4)}{V_o} \qquad (5)$$

From this, we calculate the dynamic stiffness (spring rate) of the shock mounts required to provide the desired protection.

$$K' = \frac{(f_n)^2 W}{9.8} \qquad (6)$$

Where K' = dynamic stiffness of mount(s) (lb/in)
f_n = system natural frequency (Hz)
W = supported weight

The above is the basic analysis conducted for the less involved shipping container applications. It is based on several assumptions:

• The support structure is infinitely rigid.

• There is no rebound of the container from the impact surface.

- There is no damping in the system.
- The mounted unit does not rotate.
- Shock mount stiffnesses are linear in the working range of deflection.

These same assumptions are carried through the remainder of this discussion. The first three tend in the direction of making the analysis conservative. The last assumption is one which must be watched closely based on mount size, shock levels, and installation geometry.

Shipping Container Mount Descriptions

The great majority of elastomeric (rubber) shipping container mounts are of a “sandwich” type construction. That is, there are typically two flat plates, with threaded fasteners installed, which are bonded on either side of an elastomeric pad. The general construction is shown in Figure 25.

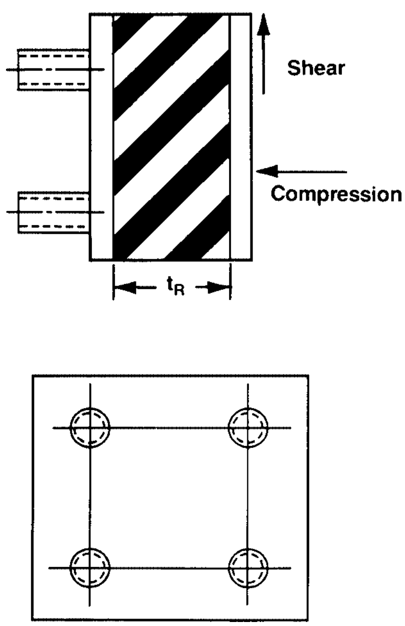


FIGURE 25
TYPICAL SHIPPING CONTAINER MOUNT CONFIGURATION

The shape of the mount can vary depending on the needs of a particular application. The standard product lines for Lord shipping container mounts are shown in the product section here.

Shipping Container Mount Stiffness

As was shown in the previous section, the stiffness of the shipping container mount determines the dynamic response of the support system. This mount stiffness depends on the geometry of the mount and the properties of the elastomer. The general equation for the shear stiffness of an elastomeric sandwich mount is:

$$K_s = \frac{AG}{t_R} \tag{7}$$

Where K_s = shear stiffness (lb/in)
A = elastomer cross-sectional area (in_2)
G = elastomer shear modulus (lb/in_2)
t_R = elastomer thickness (in)

The compression stiffness of a sandwich mount is higher than the shear stiffness by some value. This ratio of compression to shear stiffness is known as the “L” value for the mount, or:

$$L = \frac{K_C}{K_S} \tag{8}$$

Where K_C = mount compression stiffness (lb/in)
K_s = mount shear stiffness (lb/in)

The compression stiffness, like the shear stiffness, is dependent on geometry and elastomer properties. Here, the elastomer property of concern is the compression modulus. The complicating factor is that the compression modulus varies, in a nonlinear fashion, with the geometry of the mount. Figure 26 shows the general trend of the variation of compression modulus versus a geometry factor. The shape of this curve also varies with the basic hardness of the elastomer compound being used.

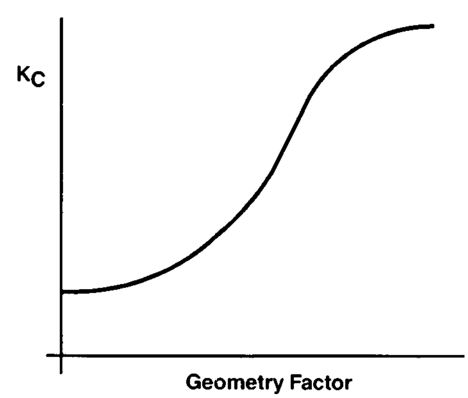


FIGURE 26
VARIATION OF COMPRESSION MODULUS
WITH GEOMETRY

It is not the intent of this guide to present mount design, but only application. Thus, let it suffice to say that, with the above background, there are specific ratios of compression to shear stiffness for various geometries for the mounts used in the shipping container industry. The "L" value is important to calculations of dynamic performance of a shipping container suspension.

The general relationship of the stiffness of the mounts, in various directions of loading, is shown schematically in the load versus deflection graph of Figure 27. It is important to note the range of linearity of the various curves. In shear, sandwich mounts can be linear up to deflections equal to 2.5 or 3.0 times the rubber thickness. In compression, this linear region may be only up to 0.25 times the rubber wall length. Shipping container mount systems assume linear stiffnesses of the mounts. Thus, care must be observed in interpreting results, particularly when compression loading of the mounts occurs.

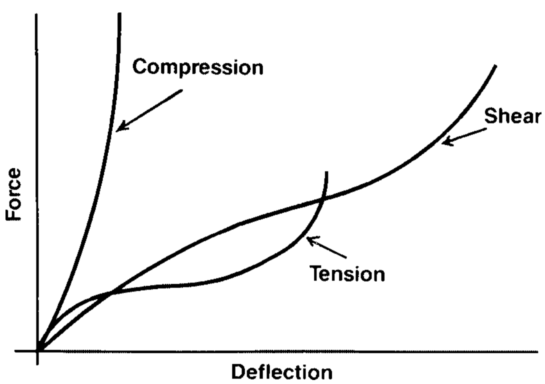


FIGURE 27
RELATIONSHIP OF VARIOUS MOUNT STIFFNESSES

Note: Mounting systems are not designed to load mounts in tension. Tension loading is to be avoided as much as possible.

In general, the best protection from shock is provided by using the mounts in a shear mode. This is not always practical nor possible as will be shown in the next section.

System Installations

Depending on system requirements, shock mounts may be installed in shipping containers in a variety of configurations. Each type of installation has a distinct response characteristic. A key concept for analyzing any shipping container mounting system is that of "elastic center."

The elastic center of a mounting system is that point in space about which the mounted equipment will rotate when subjected to an inertial load (acting through the center of gravity). The location of the elastic center of a mounting system depends on the orientation and spring rate characteristics of the mounts in the system. In most shipping container installations, the sandwich type mounts are used. This type of mount tends to project the elastic center approximately on a line extended from the compression axis. The actual point of projection depends on the "L" value of the mount being considered.

This may best be demonstrated by looking at some typical shipping container mount installations.

Simple Shear System

The simple shear system is the easiest to analyze and understand. It has some advantages to the container manufacturer in simplicity of installation, but also has some disadvantages in performance, centering on the compression stiffness characteristics of the isolator.

The simple shear installation of shock mounts is shown below.

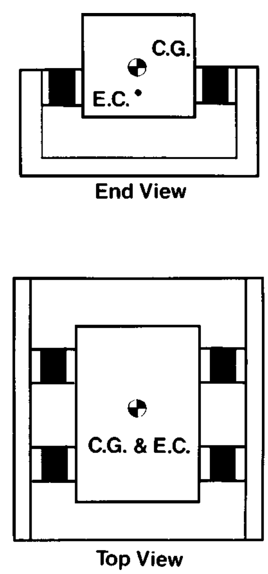


FIGURE 28
SIMPLE SHEAR MOUNTING SYSTEM

E.C. = Elastic Center of Mounting System
C.G. = Center of Gravity of Mounted Equipment

In this system, the shock mounts react loads, in the vertical and fore-aft directions, through shearing of the elastomer. This is the softest direction of the mounts and will result in the lowest accelerations transmitted to the supported equipment. Loading in the lateral direction is absorbed in compression of the mounts and rotation about the elastic center (E.C.) of the system, as shown schematically in Figure 29. This type of response is typical of side impact tests. The rotation is the result of the inertial force imposed at the center of gravity (in a shock situation) which causes an overturning moment around the system elastic center

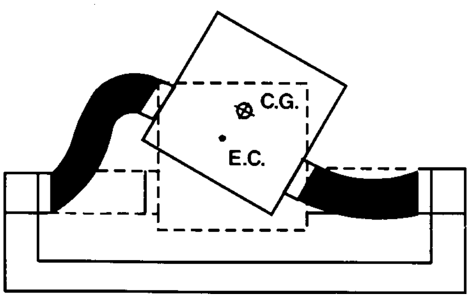


FIGURE 29
RESPONSE OF SHEAR SYSTEM TO SIDE IMPACT
(EXAGGERATED)

Focalized Systems

In some container installations, the simple shear system results in unacceptably high transmitted shock loads in the lateral direction or in unacceptably high rotational deflections at the outer edges of the mounted equipment. In such cases, "focalized" systems are often used.

The shock mounts in such systems are "focused" at some angle such that the offset between the elastic center and the center of gravity is reduced. This reduced offset lessens the overturning moments due to side impacts and, thus, results in less rotation of the mounted equipment. The compromise with a focalized system is that the mounts are not being loaded in shear; neither in the vertical direction for a semi-focalized system, nor in any axis for a fully-focalized system. This situation leads to a combination of shear and compression loading which will result in a higher effective mount stiffness and higher 'g' loads in directions that were previously shear axes. Conversely, directions that were previously compression will have a lower stiffness and will result in lower 'g' loads.

Figures 30 and 31 show semi-focalized and fully focalized systems, respectively. The semi-focalized installation has the mounts angled upward from the horizontal plane. This raises the elastic center of the mount system, increases the vertical system stiffness (due to the combination of compression and shear loading), but keeps the fore-aft axis completely in shear. The fully-focalized system places the mounts at angles up from the horizontal plane and inward toward the center of the mounted equipment. This arrangement results in combined shear and compression loading in all directions.

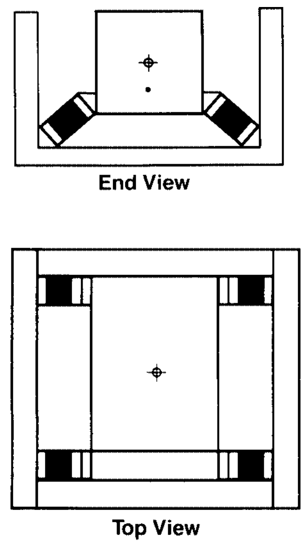


FIGURE 30
SEMI-FOCALIZED SHIPPING CONTAINER
MOUNT SYSTEM

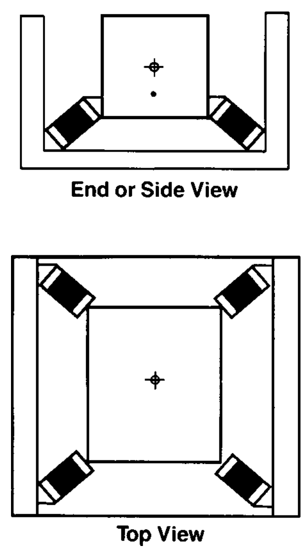


FIGURE 31
FULL-FOCALIZED SHIPPING CONTAINER
MOUNT SYSTEM

Low Fragility

Some types of equipment are more fragile than others and require better protection in their shipping containers. If the required protection cannot be achieved through the use of any of the previously described mount systems, then something special must be done. There are two basic options. First, standard sandwich mounts may be used in a gimballed arrangement. Second, a special mount design may be conceived to provide low spring rates and high deflections in all directions.

The gimballed system is shown in Figure 32. This system will use more mounts and will require considerable space for mounts, but it does have the advantage of using available mount geometries. The special design option will be more compact but has the disadvantages of development time and lack of availability.

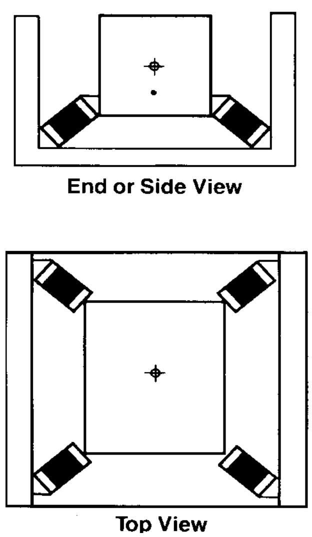


FIGURE 32
GIMBALLED MOUNTING SYSTEM

Caution: When analyzing low fragility systems, special consideration must be given to the system natural frequency. The system natural frequency must always be calculated and checked against various system requirements. One concern with low fragility systems is that they typically require very low natural frequencies and could fall into critical vibration frequency ranges for various methods of transportation (3 to 7 Hz). Thus, a low fragility mounting system may provide excellent shock protection but it will require significant sway space and could cause system natural frequencies to fall into critical ranges. Another concern here is the large static deflection imposed on the mounts. This can, over long periods, degrade performance. In cases where a low frequency system is indicated, the designer is encouraged to contact Lord.

Properties of Elastomers

The “spring” portion of typical shipping container mountings is an elastomer (rubber) specially compounded and processed to provide certain stiffness characteristics. The standard line of Lord shipping container mountings uses a specially compounded synthetic elastomer which is called “SPE®I”. This material has high strength, medium damping and good low temperature flexibility - all of which are important to shipping container use.

Besides SPE®I, other elastomers can be used but are less suited to the job at hand. For example, natural rubber has excellent strength but is not a good candidate where very low temperature performance or damping are required. Neoprene, another elastomer which has been used in some past shipping containers, is not recommended for low temperature applications.

A brief discussion of some of the properties of SPE I® elastomer will give background in the behavior of elastomeric shock mountings.

Stiffness Versus Temperature

Figure 33 shows the trends of elastomer stiffness versus temperature for typical SPE®I elastomer, Natural Rubber, and Neoprene compounds. The data on which these curves are based were compiled using low amplitude motions across standard samples of the various elastomers. It is immediately obvious that the SPE I elastomer material is far superior to typical ranges of operation for shipping containers. This is the basic reason that Lord standardized on the SPE I elastomer for shipping container mounts.

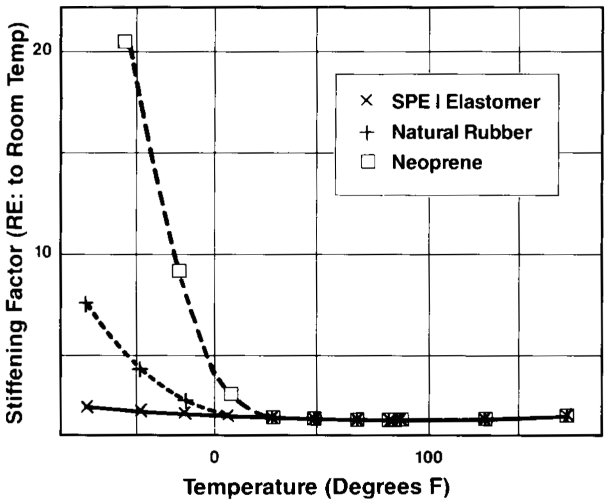


FIGURE 33
DYNAMIC STIFFNESS OF
ELASTOMERS VERSUS TEMPERATURE

Even more important is the fact that the variations in stiffness with temperature, as shown in Figure 33, must be taken into account when analyzing a shipping container installation. At low temperatures, the system natural frequencies and transmitted accelerations will be higher than at room temperature. At high temperatures, the natural frequencies and transmitted accelerations will be lower than at room temperature — provided there is enough space in the container for the system to deflect without bottoming.

Stiffness Versus Strain

Along with variations in stiffness with temperature, elastomers also exhibit different stiffnesses at different strain levels. At low strain levels, elastomers are stiffer than at high strain levels. Strain is defined as the deflection across the elastomer divided by the thickness of the elastomer

The reason for this "strain sensitivity" of elastomers lies in the molecular structure of the material. Typically the more complex the molecular structure, the higher the damping in the compound, the more pronounced the strain sensitivity will be.

The importance of this subject to the analysis of a shipping container suspension is that it must be recognized that an elastomeric shipping container mount will exhibit different stiffnesses when tested under different conditions. In general, under shock an elastomeric mount will be stiffer than when it is tested statically (with a slowly applied load). Further, an elastomeric mount will generally be stiffer still under most vibration tests than it is under shock conditions. As a rule of thumb, then it should be remembered that:

$$K_{vib} > K_{shock} > K_{static}$$

Where "K" is spring rate (stiffness) of the part.

Figure 34 shows the change in stiffness of a typical SPE I elastomer versus strain. Such a curve may be used to roughly estimate shock mount stiffness when the dynamic conditions imposed on the mounts are known.

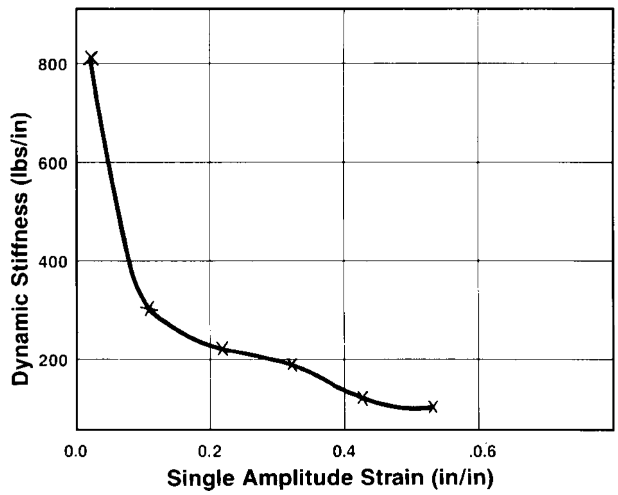


FIGURE 34
STIFFNESS VERSUS STRAIN—
TYPICAL SPE®I ELASTOMER

Drift

Elastomeric mounts under load will drift and increase their static deflection with time. This characteristic must be understood and taken into account when planning the amount of necessary sway space in a shipping container.

The total deflection to be planned for must include static deflection, dynamic motion and drift. This latter item will depend on the amount of load on the mount, the direction of the load, and the temperature at which the mount is being loaded.

Due to the nature of the variables involved, it is difficult to generalize as to the drift characteristic. Some data are available which can be used as a guideline. A typical curve is shown in Figure 35.

Figure 35 shows room temperature and elevated temperature (+158°F) drift curves for a medium stiffness SPE®I elastomer sample loaded at a static stress level of 30 psi. The shape of the curve is typical of elastomeric drift. The greatest percentage of drift occurs within the first 2 to 3 days after the load is applied. After that, the rate of drift slows asymptotically. Thus, some estimate of total drift can usually be made and included in calculations of necessary sway space.

The vertical axis of Figure 35 is in "Percent of Room Temperature Initial Deflection." Thus, for example, if a system deflects 1.0 inch under its initial load at room temperature, it may be expected to deflect another 0.80 inch (approximately) after one month at room temperature, under a constant static load. This extra deflection must be allowed for in the internal sizing of the shipping container.

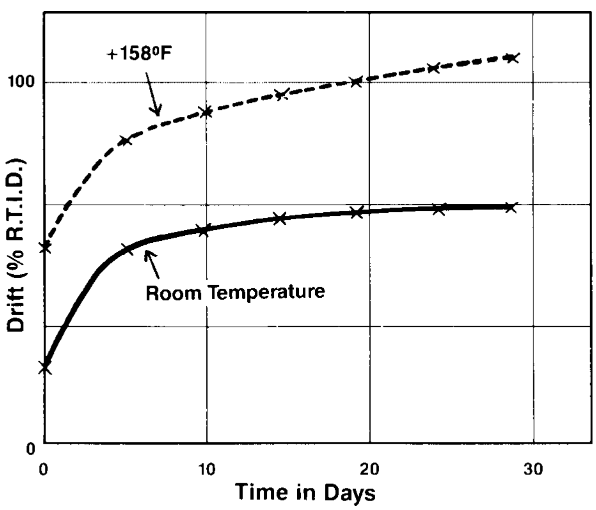


FIGURE 35
TYPICAL DRIFT CURVE—SPE® I ELASTOMER (30 PSI)

SYSTEM ANALYSES

The following section gives a basic method for analyzing most simple shipping container shock conditions. The following is based on several assumptions which must be kept in mind:

1. The properties of the shock mounts are assumed to be linear,
2. The container and mounted unit are inelastic (infinitely rigid),
3. The velocity change of the moving container is instantaneous upon impact,
4. All kinetic energy is stored in the mounts—no energy is dissipated,
5. The system is uncoupled in all directions for flat bottom and edgewise drops, and
6. For a flat side drop, the effects of phase relationship between translational and rotational modes are neglected. They are assumed in phase, which covers the worst case.

As a rule of thumb for these simplified analyses, the effects of coupling are considered minimal if the eccentricity (e) of the center of gravity from the elastic center is one third, or less, of the shortest distance between mounts. This applies providing the unit is nearly symmetrical and homogeneous.

See page 41 for list of symbols as used below

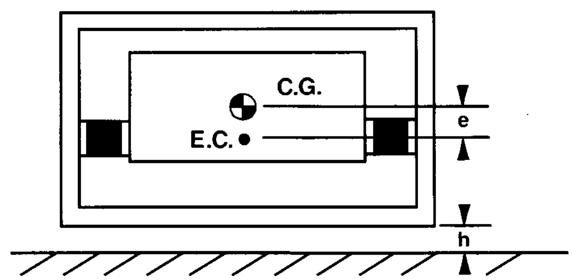


FIGURE 36
FLAT BOTTOM DROP

1) Calculate the maximum deflection required

$$d = \frac{2h}{G_o - 2}$$

2) Calculate the drop energy

PE = Wh when d - 0.1h
PE = W(h + d/2) when d ž 0.1h
and KE = PE

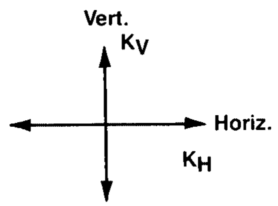
This energy must be stored in the mounts.

3) Calculate the system dynamic spring rate

$$K_V = \frac{2(KE)}{d^2}$$

$K_v \cong K_{VS}$ for natural rubber and neoprene
$K_v \cong 1.3\ K_{VS}$ for SPE I elastomer

NOTE: These relationships are valid when strains are approximately 100% or greater.



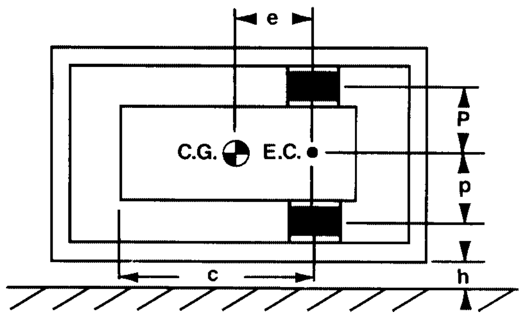


FIGURE 37
COUPLED FLAT SIDE DROP

1) Calculate deflection required for linear uncoupled system:

$$d = \frac{2h}{G_o - 2}$$

2) Calculate drop energy:

PE = Wh when d < 0.1h
PE = W(h+d/2) when d > 0.1h

NOTE: Using d/2 gives approximation of CG deflection of coupled system.

This energy must be stored in the mounts.

Thus, KE = PE

3) Calculate translational and static rotational deflection:

$$d_{ST} = W/K_v = \text{deflection @ 1g}$$

$$d_{RST} = We/K_R$$

NOTE: $K_R = K_H P^2$

4) Total energy equation is: (1g condition)

$$KE_1 = \frac{K_v d^2_{ST}}{2} + \frac{K_R d^2_{RST}}{2}$$

or

$$KE_1 = \frac{W^2}{2K_v} + \frac{W^2 e^2}{2K_H P^2}$$

5) Total acceleration at CG is approximately:

$$G \cong \sqrt{\frac{KE}{KE_1}}$$

6) G load calculated is for CG location only since moment equals weight times eccentricity (e) in the solution. Loads at points closer to EC than CG will be greater than G.

7) Calculate deflection

a) CG deflection = $d + eG(d_{RST})$

b) Top deflection = $d + cG(d_{RST})$

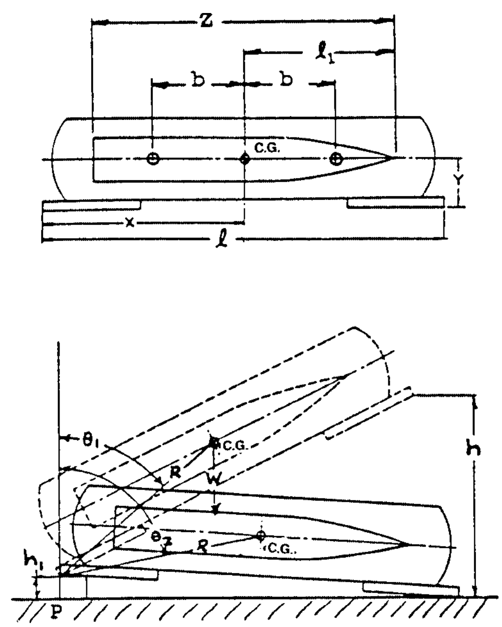


FIGURE 38
EDGEWISE ROTATIONAL END DROP ANALYSIS

A) General System Parameters:

1) Calculate: $\mathbf{R} = \sqrt{\mathbf{X}^2 + \mathbf{Y}^2}$

2) Pitch moment about point P: $\mathbf{Ip = I_{CG} + MR^2}$

3) Radius of gyration about point P: $\mathbf{r} = \sqrt{\mathbf{I_p / M}}$

4) Angles of Figure 38:

$$\theta_1 = 90^\circ \text{ - arctan y/x - arcsin } \frac{h - h_1}{\ell}$$

$$\theta_2 = 90^\circ \text{ - arctan y/x + arcsin } h_1 / \ell$$

5) Angular velocity @ impact:

$$\omega_o = \sqrt{\frac{2Rg(\cos\theta_1 - \cos\theta_2)}{r^2}}$$

6) Linear velocity of C.G. normal to container base:

$$V = \omega_1 R \cos\left(\arctan\frac{Y}{X}\right)$$

7) Linear velocity of unit end due to rotation about C.G., normal to container base:

$$V_1 = \omega_o \ell_1$$

8) Considering desired $\mathbf{G_T}$ is known, A and B must be estimated to continue with analysis

Generally a scalar sum of **A + B** is made equal to $\mathbf{G_T}$**.** Then, $\mathbf{A + B = 386G_T}$

For softer systems, i.e., $\mathbf{G_T = 10}$ or less, it is desirable to maintain a ratio of **A/B = 1** or **A = B**

Therefore, $\mathbf{A = G_T/2}$ and $\mathbf{B = G_T/2}$

B) System Response in Translation

1) Vertical translational circular frequency:

$$\omega_1 = A / V$$

2) Vertical dynamic spring rate: $K_V = (\omega_1)^2 M$

C) System Response in Rotation

1) Rotational circular frequency about C.G. is:

$$\omega_2 = B / V_1$$

2) Rotational dynamic spring rate: $K_R = (\omega_2)^2 I_{CG}$

3) Mounting spacing: $b^2 = K_R / K_V$

D) Total System Response

$$G_T = \frac{A}{g}\sin\omega_1 t + \frac{B}{g}\sin\omega_2 t \quad (t_o \text{ is } t@\text{impact} = 0)$$

$$d_T = \frac{A}{(\omega_1)^2}\sin\omega_1 t + \sin\frac{B}{(\omega 2)^2}\sin\omega_2 t$$

t = time when G_T is maximum. This is also value of t where d_T and d_M are maximum.

$$d_M = \frac{A}{(\omega_1)^2}\sin\omega_1 t + \frac{b}{\ell_1}\frac{B}{(\omega_2)^2}\sin\omega_2 t$$

Note: If ω_1 and ω_2 are very close together then:

$$G_T = A/g + B/g$$

$$d_T = A/(\omega_1)^2 + B/(\omega_2)^2$$

$$d_M = A/(\omega_1)^2 + [b/\ell_1][B/(\omega_2)^2]$$

At this point overall balance and practical design of the system must be considered.

1) Relationship of b to z and ℓ_1

2) Comparison of ω_1 and ω_2 and A and B (well balanced system has $\omega_1 \cong \omega_2$ and $A \cong B$ if possible)

E) Mounting Calculations

1) Mounting dynamic vertical spring rate: $\mathbf{k_S = K_V/n}$ **where n = number of equally loaded mounts.**

2) Mounting static vertical spring rate:

a) $K_V \cong k_S$ for natural rubber and neoprene
b) $K_V \cong k_S/1.3$ for SPE®I elastomer

NOTE: a) and b) are valid for strain values of 100% or greater

3) Mounting is selected on the following basis:
   a) Static spring rate
   b) Deflection capability (linearity and strain)
   c) Shear area (stress)
   d) Fatigue
   e) Material (special properties, i.e., temperature, etc.)

F) Container Clearance

1) Total clearance is found by considering dynamic deflection, permanent set and safety factor

   a) Total clearance for SPE I elastomer mountings

$$d_T + \frac{d_T + 2}{8} + .5 \text{ in}$$

(.5 in. is a maximum set normally encountered in SPE I mountings)

b) Total clearance for rubber or neoprene

$$= d_T + \frac{d_T + 2}{8}$$

NOTE: For temperature sensitive elastomer, total clearance should be based on high temperature performance.

COMBINATION AND OBLIQUE DROPS

Cornerwise Rotational End Drop

Analyze same as edgewise rotational end drop.

Cornerwise Drop

Calculate same as flat drop. Be certain to avoid “pure” compression loading on mounts. Offset mounts from plane through C.G. and corner to induce rotation upon impact.

Incline Impact or Pendulum Impact

Analyze as flat side drop using drop height equal to vertical rise of C.G. about point of impact. The following formula may be used.

$$d = \frac{2h}{G_o} \text{ and } PE = Wh$$

Tip Over - Roll Over

Analyze as edgewise rotational drop for side to bottom or side to top and as equivalent flat side drop for bottom to side or top to side. (Cylindrical containers should be designed to include roll-over flanges — no analysis is applicable.)

Coupled Systems

When the elastic center and center of gravity of a mounted system do not coincide, the system will, under dynamic excitation, exhibit combinations of translational and rotational modes. There are two ways of looking at this situation.

First, the system can be used as is and the rotational natural frequency calculated to determine if there is any reason for concern related to the dynamic environment to be encountered. Second, if it is determined that coupling, rotation, of the system cannot be tolerated, then the focalization angles for the mounts may be calculated to reduce or eliminate rocking of the mounted unit. The analyses of both of these cases depend on the geometry of the mounted system and the characteristics of the mounts.

The following sections show the calculations for the above cases.

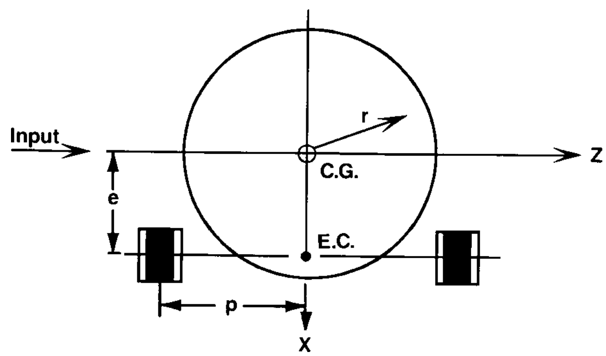


FIGURE 39
CALCULATION OF COUPLED NATURAL FREQUENCIES

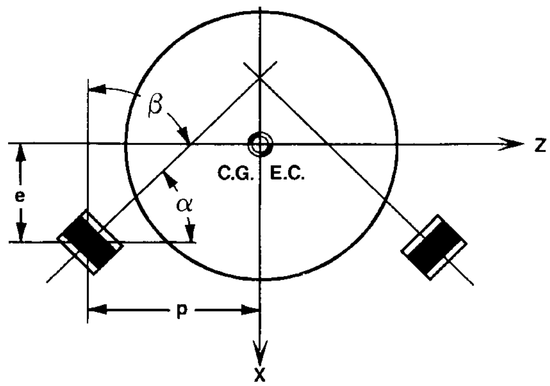
$$K_R = K_v p^2$$

$$S = \sqrt{K_R / K_H} = \sqrt{K_v p^2 / K_H}$$

$$\frac{f_c^2}{f_n^2} = 1/2\left(1 + \frac{S^2}{r^2} + \frac{e^2}{r^2}\right) \pm \sqrt{1/4\left(1 + \frac{S^2}{r^2} + \frac{e^2}{r^2}\right)^2 - \frac{S^2}{r^2}}$$

Results in two coupled natural frequencies (f_c)

NOTE: For fore and aft input, use b (1/2 mount spread, Fig. 38) in place of p, fore and aft spring rate in place of K_H, and pitch radius of gyration.



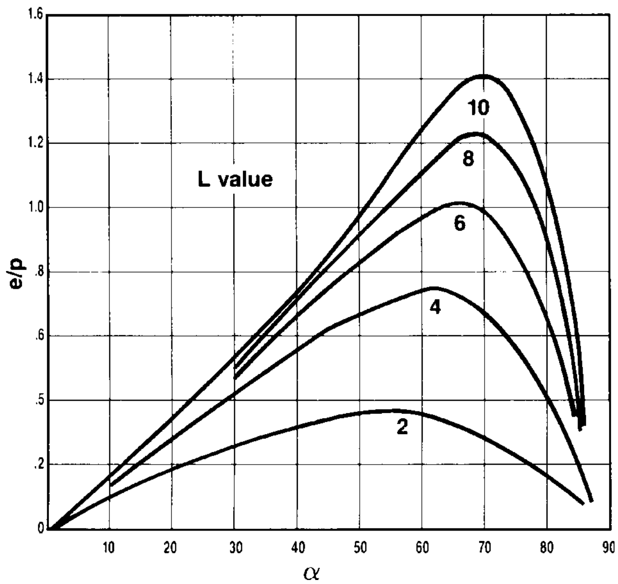


FIGURE 40
CALCULATION OF FOCALIZATION ANGLE
TO PROJECT ELASTIC CENTER TO POINT OF C.G. TO
UNCOUPLE SYSTEM

L value - Ratio of Compression to Shear Spring Rate

$$K_V = 4K_S[L\cos^2\beta + \sin^2\beta]$$

$$K_H = 4K_S[L\cos^2\alpha + \sin^2\alpha]$$

NOTE: Above analysis assumes system uses 4 mounts.

Vibration Testing

The preceding analyses have been focused on shock (drop) testing of shipping containers. Most shipping containers must also be exposed to some vibration testing and a review of critical frequencies should be made.

The key here is to recognize that the stiffness of an elastomeric isolator will typically be higher during vibration testing than during a shock or static test. The amount of stiffening depends on the magnitude of the vibration, which translates into strain across the elastomer.

The strain, during a vibration test may be calculated roughly as:

$$\in = (x_i)(T)/t_R$$

Where $\in$ **= strain (in/in)**
x_i **= single amplitude input vibration level (in)**
T = resonant transmissibility (assume 5 for SPE®I elastomer)
t_R **= thickness of elastomer (in)**

Once the dynamic strain is calculated, Figure 34 may be used to estimate the dynamic stiffness, versus the static stiffness of the mount. Then, the system natural frequencies may be calculated using the analysis previously presented.

If a resonant dwell vibration test is to be conducted, it is normal to run the test intermittently to avoid over-heating the elastomeric mounts due to hysteretic heating. The surface temperature of the mount should not be allowed to exceed +115· F.

Mount Selection

Once the dynamic analyses are completed, the required mount stiffness is known and the appropriate mount may be selected. This selection will be based on stiffness, maximum stress, and maximum strain. The following guidelines are applicable to Lord SPE®I elastomer shock mounts:

a) Maximum dynamic stress should be limited to 225 psi or less. The analysis of the most severe shock at the lowest operational temperature will result in the highest dynamic load.

b) Maximum static 1g stress should be limited to 25 psi or less.

c) Maximum dynamic strain should be 250%. The analysis of the most severe shock at the highest operational temperature will result in the highest dynamic strain.

Standard Mounts

The product section contains the standard sizes of shipping container mounts manufactured by Lord using SPE I elastomer. Wherever possible, these mounts should be used. They were selected based on years of usage data for many shipping container applications.

DATA REQUIRED FOR SHIPPING CONTAINER ANALYSES

As with any engineering problem, the quality and accuracy of the calculated solution is only as good as the information provided as input to the analysis. A Suspension System Questionnaire is available to outline the mimimum data needed for a reasonable shipping container analysis. This questionnaire, found in this catalog, can be used as a check list for self-analysis or for transmittal to Lord for a formal system analysis.

SYMBOLS

Symbol	Description	Units
a	Normal instantaneous acceleration of unit at C.G.	in/sec^2
A	Maximum vertical acceleration at the center of gravity	in/sec^2
b	Longitudinal horizontal distance from C.G. to mount (half mount spread)	inches
B	Maximum vertical acceleration at unit end due to rotation about elastic center (E.C.)	in/sec^2
c	Distance from elastic center to top of equipment	inches
CG	Center of gravity	—
d	Dynamic deflection	inches
dyn	Dynamic	—
d_M	Dynamic deflection at mount	inches
d_R	Rotational deflection	radians
d_{RST}	Static rotational deflection	radians
d_{ST}	Static deflection	inches
d_T	Deflection total at end of unit	inches
D_1	Maximum vertical deflection at C.G.	inches
D_2	Maximum vertical deflection at end of unit due to rotation about elastic center	inches
E	Eccentricity, or distance between E.C. and C.G.	inches
E_C	Elastic center	—
f_n	Natural frequency, translational	H_Z
f_C	Coupled natural frequency	H_Z
G_1	Maximum vertical acceleration at C.G.	multiples of g
G_2	Maximum vertical acceleration due to rotation at end of unit	multiples of g
G_O	Fragility of unit at C.G.	multiples of g
G_T	Total vertical acceleration at end of container	multiples of g
g	Acceleration of gravity	386 in/sec^2
h	Height of drop	inches
h_1	Vertical distance of pivot point above floor	inches
I_{CG}	Moment of inertia about C.G.	lb-in-sec^2
I_P	Moment of inertia about container pivot point	lb-in-sec^2
k	Static spring rate (single mount)	lbs/in
k_C	Dynamic compression spring rate (single mount)	lbs/in
k_S	Dynamic shear spring rate (single mount)	lbs/in
K_H	System dynamic horizontal spring rate	lbs/in

SYMBOLS

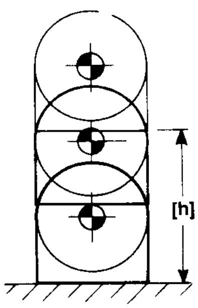
Symbol	Description	Units
K_R	System dynamic torsional or rotation spring rate	in-lbs/radian
K_T	System dynamic tension spring rate	lbs/in
K_V	System dynamic vertical spring rate	lbs/in
K_{VS}	System static vertical spring rate	lbs/in
KE	Kinetic energy	in-lbs
ℓ	Length of container, overall	inches
ℓ_1	Distance from C.G. to end of unit	inches
L	Ratio of compression stiffness to shear stiffness	—
M	Mass of equipment	lb-sec^2/in
p	Lateral horizontal distance from C.G. to mount (half mount spread)	inches
PE	Potential energy	in-lbs
r	Radius of gyration	inches
R	Distance from container pivot point to C.G.	inches
S	Square root of ratio of rotational spring rate to lateral translation spring rate	inches
St	Static	—
t	Time	seconds
V	Normal linear velocity of C.G. at impact	in/sec
V_1	Normal linear velocity of unit end due to rotation about elastic center	in/sec
W	Weight of suspended mass	lbs
X	Horizontal distance from container pivot point (p) to unit C.G.	inches
Y	Vertical distance from container pivot point (p) to unit C.G.	inches
Z	Length of suspended unit	inches
α	Angle between the compression axis and horizontal	degrees
β	Angle between the compression axis and vertical	degrees
θ_1	Angle between a line joining C.G. and pivot point (p) and vertical before drop (when $h_1 = 0$)	degrees
θ_2	Angle between a line joining C.G. and pivot point (p) and vertical after drop (when $h_1 = 0$)	degrees
ω_0	Angular velocity of C.G. at impact	rad/sec
ω_1	Vertical translational circular natural frequency	rad/sec
ω_2	Rotational circular natural frequency	rad/sec

STANDARD SHIPPING CONTAINER SHOCK TESTS

No matter what mode of transportation is used, shock represents the most serious threat to equipment reliability. The standard tests described here are intended to simulate the worst shock conditions that would be expected for shipping/handling environments. Selected tests from those shown here are included in packaging specifications and used for designing shipping container suspension systems.

The letter "h" in the diagrams depicts the drop height specified in the applicable packaging specification. Exceptions: in Test 7 and 11 an impact velocity will be specified; in Test 9 and 10 neither drop height nor velocity is specified.

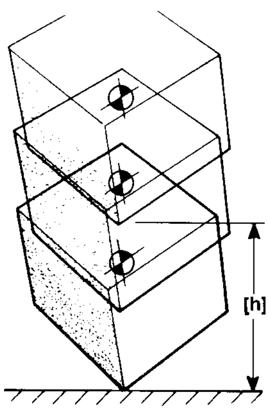
Test 1



Flat Drop

Container shall be raised the specified vertical distance and allowed to fall freely to a concrete or similarly hard surface so that container strikes flat on the skids or surface involved.

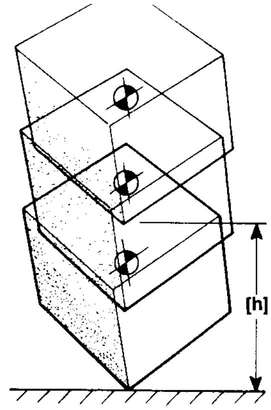
Test 2



Corner Drop

Container shall be raised the specified vertical distance such that the container is suspended with the center of gravity vertically above the striking corner. Container shall be allowed to fall freely to a concrete or similarly hard surface, striking corner first. Cylindrical containers shall be dropped on each quarter.

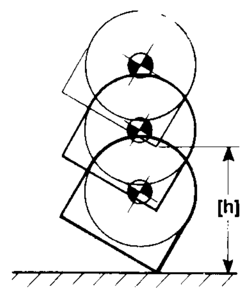
Test 3



Corner Drop (special)

Container shall be raised the specified vertical distance so that it will strike at the greatest angle possible, still ensuring that the container will come to rest on its base. The test shall be repeated for each of the corners or quarters.

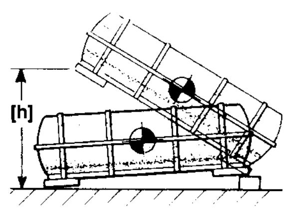
Test 4



Edge Drop

Container shall be raised the specified vertical distance, such that the container is suspended with the center of gravity vertically above the striking edge. The container shall be allowed to fall freely to a concrete or similarly hard surface, striking edge first.

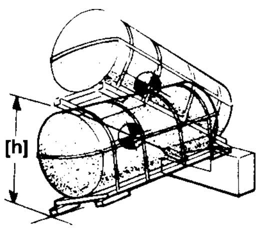
Test 5



Edgewise Rotational Drop

Container shall be supported at one end of the base on a sill or block of specified height and at right angles to skids. The opposite end shall be raised to the specified vertical height and allowed to fall freely onto a concrete or similarly hard surface. If container size and center of gravity location prevent dropping from prescribed height, the greatest attainable height shall be the height of the drops.

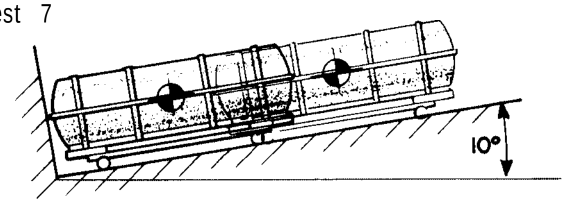
Test 6



Cornerwise Rotational Drop

Container shall be supported at one corner of its base on a low sill or block of specified height. The other corner of the same end shall be supported by a higher sill or block. The lowest point of the opposite end shall be raised to the specified vertical height and allowed to fall freely onto a concrete or similarly hard surface. If container size and center of gravity location prevent dropping from prescribed height, the greatest attainable height shall be the height of the drops.

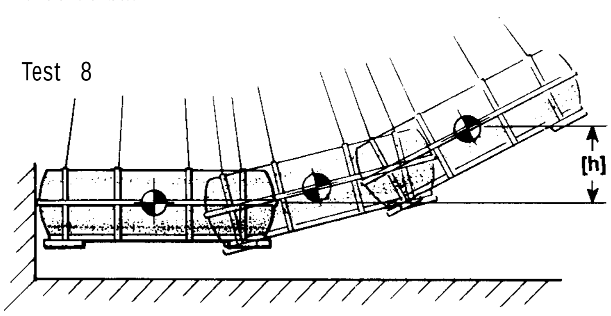
Test 7



Inclined Impact

Test shall be in accordance with ASTM Standard Method D880, "The Inclined Impact Test for Shipping Containers," suitably modified to accommodate the container. Velocity at impact shall be as specified. The Pendulum Impact may be used in lieu of this test, and vice versa.

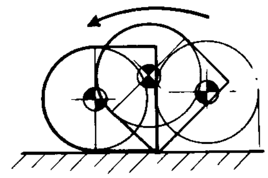
Test 8



Pedulum Impact

Container shall be suspended by 4 or more ropes or cables 16 feet or more long. Container shall be pulled back so that the center of gravity has been raised the specified distance. Container shall be released, allowing the end surface or skid, whichever extends further, to strike on an unyielding barrier of concrete or similarly hard material that is perpendicular to the container at impact.

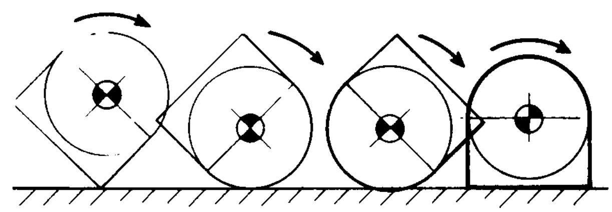
Test 9



Tip Over Test

Container, erect on its base, shall be slowly tipped (in the direction specified) until it falls freely and solely by its own weight to a concrete or similarly hard floor.

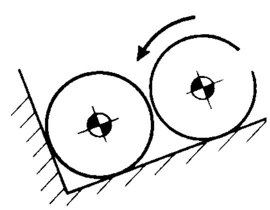
Test 10



Rollover Test

Container, erect on its base, shall be tipped sideways until it falls freely and solely of its own weight to a concrete or similarly hard surface. This shall be repeated with falls from the side to top, from top to the other side, and from other side to the base, thus completing one revolution.

Test 11



Rolling Impact Test (cylindrical containers)

Container shall be allowed to roll down an incline on its rolling flanges and shall strike a vertical, rigid, flat surface at a specified velocity.

An Invitation

The numerous isolators presented in this catalog have been designed to cover a wide range of aerospace vibration and shock isolation problems. If there are questions concerning any of these products or this catalog, or if there is need of assistance for particularly difficult installations, do not hestitate to contact Lord. See page 103 for contact information. Many years of experience may be brought to the task to provide an optimal solution.

Additionally, Engineering Data Sheets for electronic equipment and for shipping container applications are included. Providing as much of this information as is possible will assist in the analysis of difficult installations.