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Active Control of Stick-Slip Vibrations: The Role of Fully Coupled Dynamics

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Abstract

Oilwell drilling is often accompanied by self-excited stick-slip vibrations. This type of motion may also excite severe axial and lateral vibrations in the bottom hole assembly, causing damage to the equipment. This paper presents a fully coupled model for axial, bending and torsional vibrations, and an active control strategy for stick-slip vibrations. The proposed model includes the mutual dependence of these vibrations, as well as their related effects such as, bit/formation and drillstring/borehole wall interactions. The control strategy is based on optimal state feedback control designed to control the drillstring rotational motion. Simulation results are in close qualitative agreement with field observations regarding stick-slip vibrations. It is shown that the proposed control is effective in suppressing stick-slip vibrations once they are initiated. It is also demonstrated, that axial vibrations help in reducing stick-slip vibrations and the control effort. However, care must be taken in selecting a set of operating parameters to avoid transient instabilities in the axial and lateral motions.

Introduction

It is well known that stick-slip vibrations are detrimental to the service life of oilwell drillstrings and down-hole equipment. Large cyclic stresses induced by this type of motion can lead to fatigue problems. In addition, the high bit speed level in the slip phase can excite severe axial and lateral vibrations in the bottom hole assembly, which may cause bit bounce, excessive bit wear and reduction in the penetration rate. Stick-slip vibrations are self-excited, and generally disappear as the rotary table speed is increased beyond a threshold value. However, increasing the rotary speed may cause lateral problems such as backward and forward whirling, impacts with the borehole wall and parametric instabilities. Therefore, it is desirable to extend the range of safe drilling speeds. In order to achieve this, a proper understanding of the coupled dynamics of drillstrings is necessary. For this reason drillstring vibrations and ways to control them have received a lot of attention in recent years [1-8].

The control methods include operational guidelines to avoid, eliminate or reduce torsional vibrations as well as active control methods using feedback. Although most proposed control methods have been shown to be successful in controlling torsional vibrations, the effects of this control on bending vibrations have not been studied. In order to design and implement an effective control system, a coupled model is essential for identifying the critical speeds as well as predicting the behavior of the whole system [9]. Recently, the authors proposed a model that considers the full coupling between torsional and lateral vibrations of actively controlled drillstrings [10]. The proposed model was demonstrated to be quite realistic with respect to stick-slip vibrations, which were effectively eliminated through an optimal state feedback control scheme. In the current paper, this model is extended to include the effects of axial motion. The Weight-on-Bit (WOB) and Torque-on-Bit (TOB) expressions are more realistic since they are directly related to the axial and torsional motions. Simulation results show that axial vibrations have a positive effect in reducing stick-slip vibrations and the control effort. However, control of torsional vibrations may have negative effects in increasing axial and lateral vibrations. Therefore, care must be taken in selecting the operating parameters.

Fully Coupled Dynamics

A rotary drilling system is used for drilling deep wells for oil or gas production. The equations of motion of such system are obtained by using a simplified lumped parameter model. The essential components of the system and the necessary geometry used for the model are shown in Fig. 1. The equations originally developed in [10] are extended to include the axial motion. The Weight-on-Bit (WOB) is now obtained from the contact characteristics between the bit and the formation, which is more realistic than just using a prescribed function. The resulting coupled equations are given as

$$\begin{aligned}&(m+m_f)(\ddot{r}-r\dot{\theta}^2)+k(x,\phi,\dot{\phi})r+c_h|\mathrm{v}|\dot{r}\\&=(m+m_f)e_0[\dot{\phi}^2\cos(\phi-\theta)+\ddot{\phi}\sin(\phi-\theta)]-F_r\end{aligned} \quad \text{……..(1)}$$

$$(m+m_f)(r\ddot{\theta}+2\dot{r}\dot{\theta})+c_h|\mathrm{v}|r\dot{\theta} = (m+m_f)e_0[\dot{\phi}^2 \sin(\phi-\theta)-\ddot{\phi}\cos(\phi-\theta)]+F_\theta \quad \text{.........(2)}$$

$$J\ddot{\phi}+k_T(\phi-\phi_{rt})+c_v\dot{\phi}+c_h|\mathrm{v}|\dot{r}e_0\sin(\phi-\theta) - c_h|\mathrm{v}|r\dot{\theta}e_0\cos(\phi-\theta) = -T(x,\phi,\dot{\phi}) + F_\theta[R-e_0\cos(\phi-\theta)]-F_r e_0\sin(\phi-\theta) \quad \text{.................(3)}$$

$$(J_{rt}+n^2J_m)\ddot{\phi}_{rt}+k_T(\phi_{rt}-\phi)+c_{rt}\dot{\phi}_{rt}-nK_mI=0 \quad \text{........(4)}$$

$$L\dot{I}+R_mI+K_mn\dot{\phi}_{rt}=V_c \quad \text{..................................(5)}$$

$$m_a\ddot{x}+c_a\dot{x}+k_ax=-F_f(x,\phi) \quad \text{....................................(6)}$$

The equivalent system parameters for the lumped model are derived from the associated continuous model of the drillstring by using a Lagrangian approach, and are given in the Appendix. In this approach, the stabilized section of the drill collars is modeled as a simply supported beam for transverse motion of the collars, and the whole drillstring is assumed to be fixed at the top, and free at the bit for the axial and torsional motion. The drill collars are assumed to be rigid for torsional vibrations, in other words, the torsional deformations are assumed to take place only in the drill pipe.

The WOB is always positive and is obtained from the bit formation interaction and is given as

$$F(x,\phi)=F_0+F_f(x,\phi) \quad \text{...(7)}$$

The fluctuating component of WOB is given from the contact condition at the bit as

$$F_f(x,\phi)=k_c(x-x_0\sin n_b\phi) \quad \text{...(8)}$$

The TOB is given as

$$T(x,\phi,\dot{\phi})=\mu F(x,\phi)f(\dot{\phi}) \quad \text{..(9)}$$

The continous function of $f(\dot{\phi})$ is used to represent the effect of bit speed on TOB and given as

$$f(\dot{\phi})=\tanh\dot{\phi}+\frac{\alpha_1\dot{\phi}}{(1+\alpha_2\dot{\phi}^2)} \quad \text{..(10)}$$

The dependence of the WOB and TOB on the bit angular displacement and velocity was initially proposed by the authors [10] as an extension of harmonic functions of time used in earlier studies. Although this assumption has provided some insight towards understanding the nature of stick-slip vibrations, it was incomplete since the effect of rate of penetration was not included. These effects are supported by earlier experimental and analytical studies [3,11-13].

With these WOB and TOB models, the bit/formation interactions are now expressed as functions of motion. As a consequence, linear phenomena such as whirling, and simple and parametric resonance, which are based on the assumption of harmonic excitations, are no longer straightforward. As WOB is a result of the contact between the bit and the formation, also other intermittent external excitations are the radial and transverse contact forces, which result from impact of the drill collars with the borehole wall. When there is no contact the associated contact forces are zero. While the bit/formation contact is modeled by a quasistatic model, the impacts between the drill collars and the borehole wall are accounted for by using the momentum balance method with appropriate coefficients of restitution and friction. This impact model is capable of capturing both rolling with and without slip of the drill collars along the borehole wall and is given in detail in Reference [10].

Reduced Order Model for Controller Design

In control design a linear model is preferred. Such a model can be obtained by linearizing the governing equations (1)-(6). In this case, the lateral and axial motions become uncontrollable with the drive motor, and need not be included in the model. Thus, the following set of equations represents the reduced order linear model, which can be used for control design:

$$J\ddot{\phi}+k_T(\phi-\phi_{rt})+c_v\dot{\phi}=0 \quad \text{...(11)}$$

$$(J_{rt}+n^2J_m)\ddot{\phi}_{rt}+k_T(\phi_{rt}-\phi)+c_{rt}\dot{\phi}_{rt}-nK_mI=0 \quad \text{.....(12)}$$

$$L\dot{I}+R_mI+K_mn\dot{\phi}_{rt}=V_c \quad \text{................................(13)}$$

It should be noted that the reduced order model is only used to design a linear controller. The resulting control is then applied to the full model to evaluate the performance of the proposed controller. The problem can be considered as a set point control problem where the objective is to bring the table and bit speeds to their desired values within a prescribed period of time when the system is disturbed by an external influence (e.g., bit torque). A Linear Quadratic Regulator (LQR) controller can then be designed. This controller is selected because it allows the designer to have a balance between the closed loop performance and the required control effort.

The control voltage necessary to keep the torsional vibrations zero while maintaining a desired bit and rotary table speed is given by

$$V_c=K_mn\omega_d-K_1(\phi-\phi_{rt})-K_2(\phi_{rt}-\omega_dt) - K_3(\dot{\phi}_{rt}-\omega_d)-K_4(\dot{\phi}-\omega_d)-K_5I \quad \text{.................(14)}$$

It is important to examine each term in the control equation. The first term is the open loop voltage to be applied in case no feedback is used. The second term is essentially a torque feedback. The third and the fourth terms are classical integral and proportional feedback terms applied to the rotary speed (PI control), the fifth term is the bit speed feedback, and the last term represents the current feedback. Except for the bit speed, all other quantities in the control can easily be measured or determined. The bit speed measurement requires downhole equipment and may be the most challenging task. It is becoming a common practice, however, to use an instrumented bit which makes this measurement possible. Even in the absence of such measurements, a state estimator can be designed to estimate the bit speed from the other measurements since it is observable through the other states.

Results and Discussion

The parameters used in the following simulations are shown in Table 1, and represent a typical case in oilwell drilling operations. The desired rotary table speed is chosen as 11.6 rad/s (110 rpm) which is within the common operating range for oilwell drilling. From a simple linear and uncoupled analysis for this setup, the critical frequency for axial and torsional resonances are found to be 3.97 rad/s, and 1.85 rad/s, repectively. The critical frequency for whirling resonance (due to bending) is found to be 6.14 rad/s. It is assumed that the rotary table and the bit are rotating at the desired speed when the bit is off bottom. When the bit starts to interact with the formation the system will inevitably be disturbed with the possibility of axial, torsional and bending vibrations. The control objective is to bring the table speed back to the desired speed while minimizing these vibrations.

Figures 2-5 show the response when the axial motion is not considered. Though the rotary table speed does not change appreciably, significant initial stick-slip behavior is seen at the bit. The bit momentarily stops causing the TOB and the top torque to reach very high values. When the bit starts slipping, the energy stored in the drillstring is released causing very large torsional and bending vibrations. The similarity between the TOB time history presented in Figure 4 and the field data observed by Halsey et al. [6] is remarkable. Therefore, the proposed model seems to represent the stick-slip behavior quite well. In addition, since the model includes the coupling between the torsional and bending vibrations, the effect of torsional vibrations are clearly seen on the bending vibrations (Figures 4 and 5). The stick-slip vibrations are the main reasons for large lateral amplitudes and the complicated trajectory of the collars. The effect of bending vibrations on the torsional motion, however, is not as significant in this case since there are no impacts with the borehole wall. Clearly, the presence of impacts will cause more significant interaction between lateral and torsional motions. The controller appears to be effective in reducing stick-slip vibrations.

Figures 6-10 shows the response when the axial motion is considered. Despite its small amplitudes (see Figure 6) the axial vibrations seem to affect the system behavior significantly. First, the axial vibrations help reduce the severity of stick-slip vibrations as seen in Figures 7 and 8. Consequently, the controller is more effective in reducing the fluctuations in the table and bit speeds. It appears that some of the energy stored during stick phase is transferred into axial vibrations, which in turn affects the magnitudes of WOB and TOB. On the other hand, with the presence of axial motion, the control of torsional vibrations may have a negative effect in increasing lateral vibrations as can be seen in Figures 9 and 10. The nature of the growth in the lateral vibrations, which led to several impacts between the collars and the borehole wall, suggest some form of transient instability which may be caused by a momentary parametric resonance. After the transient, however, the lateral vibrations are also reduced to the steady state whirling amplitudes. In any case, this simulation shows that the transient response with respect to all modes of vibrations should be investigated before making a positive decision about a particular operating condition (i.e., desired rpm, and controller gains). Figures 11 and 12 compare the WOB and the control voltage with and without axial motion. With the axial vibrations there is a possibility for bit bounce or bit lift-off when the WOB reduces to zero. The energy transferred to axial motion during the slip phase caused a large variation in the WOB around 4 s as seen in Figure 11. Since the severity of stick-slip vibrations is reduced with the presence of axial motion, the control voltage is also reduced as seen in Figure 12.

Clearly, the choice of the desired rotary table speed influences the system response. Simulations can easily be carried out on a PC at the rig, and suitable operating conditions can be obtained for efficient drilling.

Conclusions

A study of the coupled axial, torsional and bending vibrations of an actively controlled drillstring has been presented. The proposed dynamic model includes the mutual dependence of axial, torsional and bending vibrations. Furthermore, the bit/formation and drillstring/borehole wall interactions are assumed to be related to drillstring axial, torsional and lateral motions. Simulation results are in close qualitative agreement with field observations regarding stick-slip vibrations. These vibrations are self-excited, and they generally disappear as the rotary table speed is increased beyond a threshold value. However, since increasing the rotary speed may cause lateral problems such as backward and forward whirling, impacts with the borehole wall, and parametric instabilities it is desirable to extend the range of safe drilling speeds. An optimal state feedback control has been designed to control the drillstring rotational motion. It has been shown that the proposed control can be effective in suppressing stick-slip vibrations once they are initiated. Therefore, it is possible to drill at lower speeds, which are otherwise not possible without active control. Simulation results showed that axial vibrations have a positive effect in reducing stick-slip vibrations and the control effort. However, control of torsional vibrations may have negative effects in increasing axial and lateral vibrations. Therefore, care must be taken in selecting the operating parameters.

Nomenclature

C_A = added mass coefficient
c_a = effective damping for axial motion, Ns/m
C_D = drag coefficient
c_h = hydrodynamic damping coefficient, Ns^2/m^2
c_v = viscous damping coefficient, Nms
c_{rt} = equivalent viscous damping coefficient, Nms
d_h = borehole diameter, m

d_i = inside collar diameter, m
d_o = outside collar diameter, m
$\bar{d}_i$ = inside drill pipe diameter, m
$\bar{d}_o$ = outside drill pipe diameter, m
e_0 = eccentricity of the collars, m
E = Young's modulus, Pa
F = weight-on-bit (WOB), N
F_0 = static component of WOB, N
F_f = fluctuating component of WOB, N
F_h = transverse contact force, N
F_r = radial contact force, N
G = Shear modulus, Pa
I = current, A
I_A = area moment of inertia, m^4
I_p = polar area moment of inertia, m^4
J = drillstring mass moment of inertia, kgm^2
J_m = inertia of drive motor, kgm^2
J_{rt} = inertia of the rotary table, kgm^2
k = bending stiffness of collars, N/m
k_a = effective axial stiffness, N/m
k_c = contact stiffness, N/m
k_i = controller gains
K_m = motor constant, Vs
k_T = torsional stiffness, Nm/rad
L = motor inductance, H
m = effective mass of collars, kg
m_a = effective drillstring mass, kg
m_f = added fluid mass, kg
n = gear ratio
n_b = bit factor
R_c = collar radius
R_m = armature resistance, Ω
T = Torque, Nm
v = velocity of collar geometric center, m/s
V_c = control voltage, V
x_0 = amplitude of surface profile, m
α_1, α_2 = coefficients for speed factor
ϕ = drill collar angular displacement, rad
ϕ_{rt} = rotary table angular displacement, rad
μ = cutting force factor
μ_f = viscosity of drilling mud, Ns/m^2
ρ = drillstring material density, kg/m^3
ρ_f = density of drilling mud, kg/m^3
ω_d = desired rotary table speed, rad/s

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Appendix- Lumped Model Parameters

Assuming a one-mode approximation for axial, lateral and torsional vibrations, the equivalent lumped model parameters are obtained as

$$J = 2\rho I_A l_2 + (1/3)\rho I_p l_3 \quad \text{(A-1)}$$

$$m = \rho\pi(d_o^2 - d_i^2)l_1/8 \quad \text{(A-2)}$$

$$m_f = \pi\rho_f(d_i^2 + C_A d_o^2)l_1/8 \quad \text{(A-3)}$$

$$k(x,\phi,\dot{\phi}) = \frac{EI_A\pi^4}{2l_1^3} - \frac{T\pi^3}{2l_1^2} - \frac{F\pi^2}{2l_1} \quad \text{(A-4)}$$

$$k_T = \frac{GI_p}{l_3} \quad \text{(A-5)}$$

$$c_h = (2/3\pi)\rho_f C_D d_o l_1 \quad \text{(A-6)}$$

$$c_v = \frac{\pi\mu_f d_o^3 l_2}{2(d_h - d_o)} \quad \text{(A-7)}$$

$$m_a = 2(m + m_f) + \rho\pi(\bar{d}_o^2 - \bar{d}_i^2)l_3 / 12 \text{ ....(A-8)}$$

$$k_a = E\pi(\bar{d}_o^2 - \bar{d}_i^2) / 4l_3 \text{ ....(A-9)}$$

TABLE 1- Parameters used in the simulations

Drillstring	*Drilling mud*
E = 210 GPa	ρ_f = 1500 kg/m^3
ρ = 7850 kg/m^3	C_D = 1
d_o = 0.2286 m	C_A = 1.7
d_i = 0.0762 m	μ_f = 0.2 Ns/m^2
e_0 = 0.0127 m	
l_1 = 19.81 m	*Borehole*
l_2 = 200 m	
l_3 = 2000 m	E = 210 Gpa
$\bar{d}_o$ = 0.127 m	ρ = 7850 kg/m^3
$\bar{d}_i$ = 0.095 m	d_h = 0.4286 m
c_a = 0	
Rotary drive system	
J_{rt} = 930 kgm^2	R_m = 0.01 Ω
J_m = 23 kgm^2	L = 0.005 H
c_{rt} = 0	K_m = 6 Vs
n = 7.2	
Weight and torque on bit	
F_0 = 100 kN	x_0 = 0.01 m
k_c = 25000 kN/m	μ = 0.04
α_1 = 2.0	α_2 = 1.0
n_b = 1	

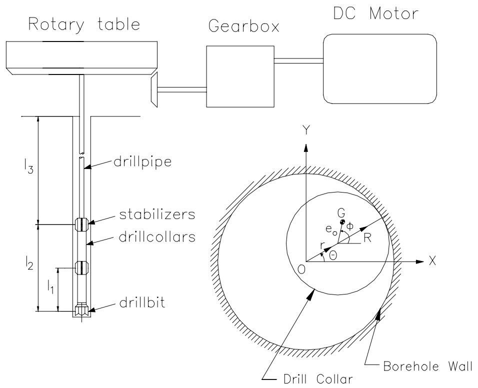

Fig. 1 – Essential components of the system and the geometry used for modeling.

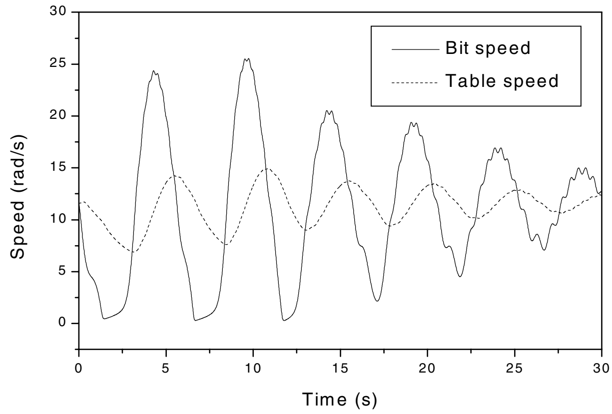

Fig. 2 – Effect of control on stick-slip vibrations as seen in the bit and table speeds (without axial motion).

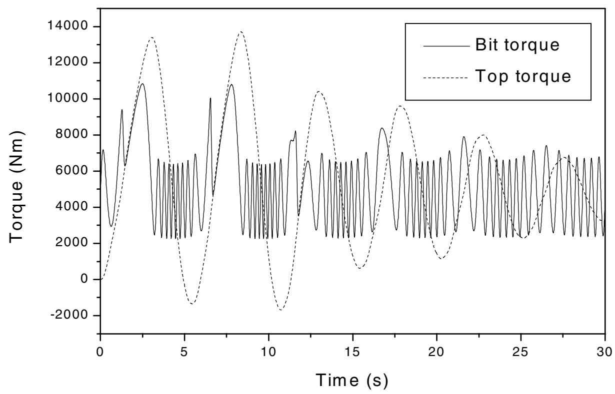

Fig. 3 – Effect of control on stick-slip vibrations as seen in the bit and top torques (without axial motion).

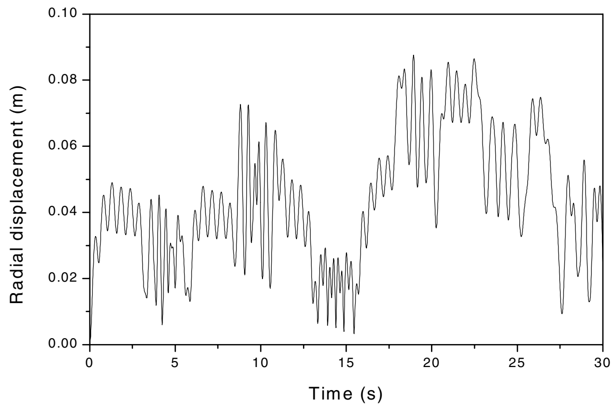

Fig. 4 – Effect of control on stick-slip vibrations as seen in the lateral motion (without axial motion).

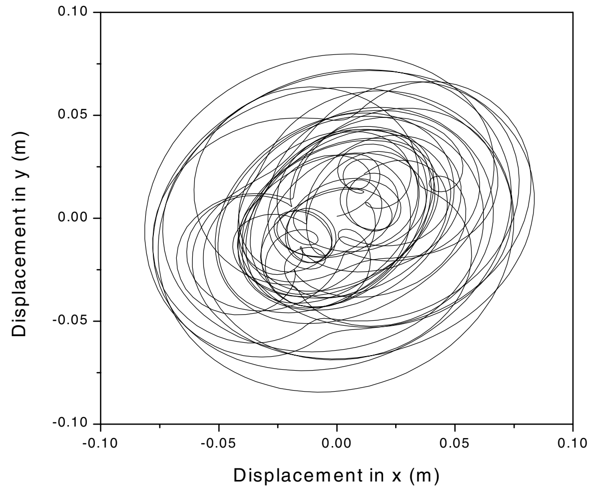

Fig. 5 – Trajectory of the geometric center of the drill collars in the case of stick-slip vibrations (without axial motion).

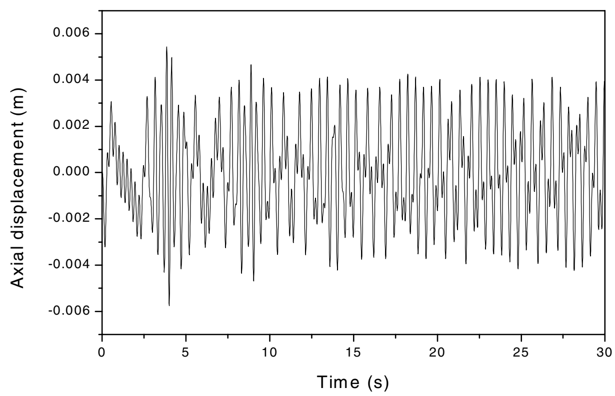

Fig. 6 – Axial vibrations at the bit.

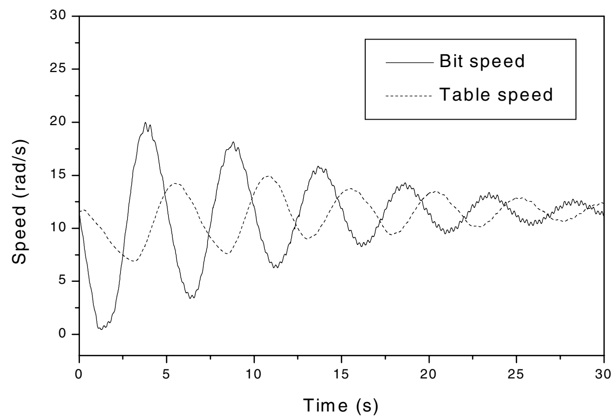

Fig. 7 – Effect of control on stick-slip vibrations as seen in the bit and table speeds (with axial motion).

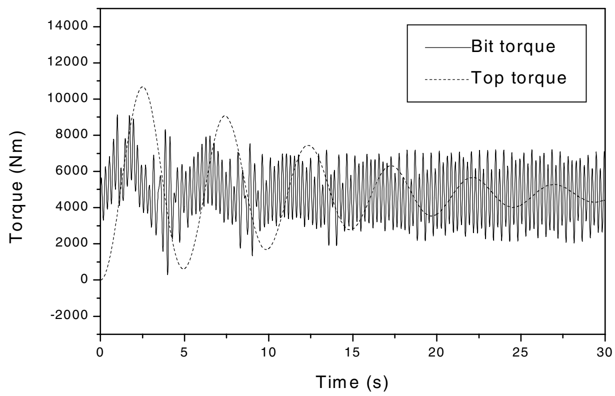

Fig. 8 – Effect of control on stick-slip vibrations as seen in the bit and top torques (with axial motion).

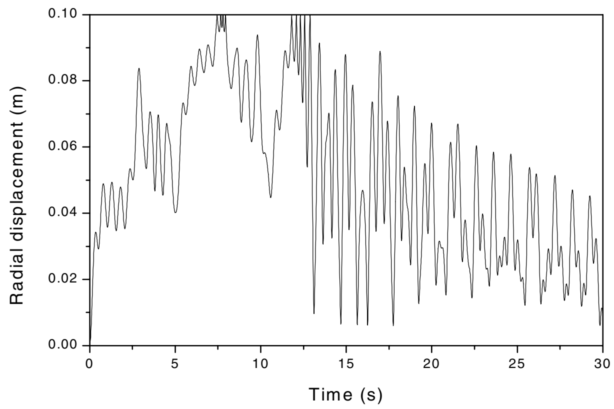

Fig. 9 – Effect of control on stick-slip vibrations as seen in the lateral motion (with axial motion).

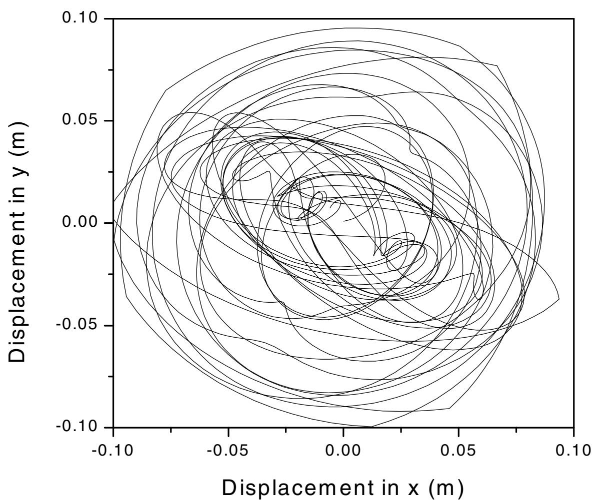

Fig. 10 – Trajectory of the geometric center of the drill collars in the case of stick-slip vibrations (with axial motion).

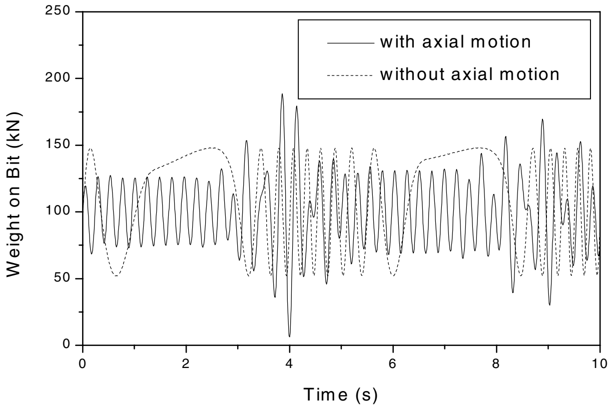

Fig. 11 – Comparison of the WOB with and without axial motion.

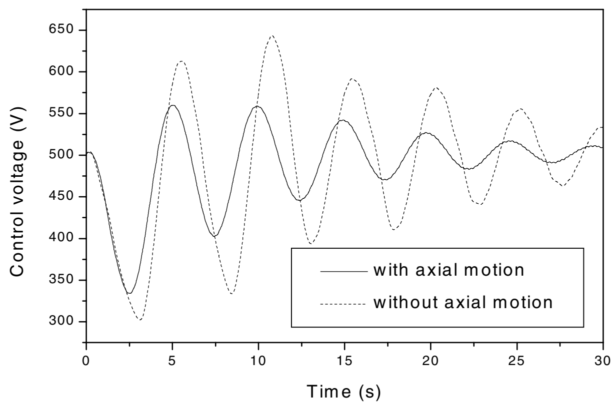

Fig. 12 – Comparison of the control voltage with and without axial motion.